REVIEW OF LIMIT EQUILIBRIUM METHODS FOR SOIL NAIL DESIGN

GEO REPORT No. 208

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PREFACE

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R.K.S. Chan
Head, Geotechnical Engineering Office
August 2007

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FOREWORD

This Report presents the results of a review study on various limit equilibrium methods commonly used for the soil nail design in Hong Kong, in an attempt to examine some issues and concerns raised by the practitioners.

Analysis of soil-nailed slopes using the programme SLOPE/W has been carried out to examine these issues. From the study, some observations and recommendations on the slope stability analysis of soil-nailed slopes using limit equilibrium methods of slices are provided.

The study was carried out by Mr Y.K. Shiu, Dr G.W.K. Chang and Dr W.M. Cheung of the Standards and Testing Division. Many other colleagues provided constructive comments on a draft of this report. Their contribution is gratefully acknowledged.

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ABSTRACT

This report attempts to address some issues and concerns about the limit equilibrium methods of slices raised by the practitioners. Stability computations have been carried out on a nailed slope and a nailed excavation using the Bishop's Simplified method, the Janbu's Simplified method, the Janbu's Generalized method and the Morgenstern-Price method. Both the nailed slope and the nailed excavation give similar computational results.

Main observations from the computational results include the followings:

(a) The Factor of Safety (FoS) values computed using the Bishop's Simplified method and the Morgenstern-Price method are sensitive to the assumed locations of nail forces.

(b) The FoS values computed using the the Janbu's Simplified method are insensitive to the assumed locations of nail forces. This is an inherent limitation of the method.

(c) If the applied resultant nail force is located above the actual resultant nail force, the Janbu's Simplified method tends to give higher FoS values than those given by the Morgenstern-Price method. The Janbu's simplified method may give a conservative or an unsafe solution depending on the nail pattern and locations.

(d) For the four methods reviewed, the FoS values derived based on the actual loading condition are close to those based on the two conditions of evenly distributed nail forces and single nail force applied at the mid-height, as the locations of the resultant force of the three loading conditions are similar.

(e) Nail forces applied on the surface of slope or excavation give FoS values very similar to those given by the forces applied on the slip surface where the lines of action intersect the slip surface. However, the inter-slice forces in the latter case may not be reasonable.

(f) All the methods have encountered convergence problems. The problems are most serious for the Janbu's Generalised method which has not produced converged solutions for any of the slips analysed.

(g) The approach of using the nail force in the same inclination of the soil nail and that of using the horizontal component of the nail force produce different FoS values. The latter ignores the vertical component of the nail force. The difference is not significant for small nail inclinations

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commonly used in design.

(h) The limit equilibrium methods have some limitations. Such limitations should be borne in mind when interpreting the results of stability computations, especially when the stress-strain assumptions may not be valid (e.g. when compressive forces are mobilized in steeply inclined nails).

This report presents the details and results of the study. It also provides recommendations on some design aspects of soil nailing using the limit equilibrium methods of slices.

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CONTENTS

1. INTRODUCTION

Limit equilibrium methods (LEM) of slices are routinely used for designing soil nails in Hong Kong. Despite the fact that these methods have been used for a long time, practitioners raise questions and concerns on the current practice from time to time. The following issues have come up lately relating to the design of soil-nailed cut slopes:

(a) Do different limit equilibrium methods give very different design results? If so, which one is correct?

(b) Does the assumption on the distribution and location of applied nail force significantly affect the result?

(c) Should the nail force be applied at the slope surface or at the location where the line of action of the force intersects the slip surface?

(d) Soil nail designs in the past generally adopted the horizontal component of the nail force in the analysis. Computer programs nowadays allow input of forces along the line of action of soil nails. Do the two methods of calculation give very different results? If so, which method should be used?

(e) The problem of non-convergence is frequently encountered in slope stability analysis.

(f) The slip surface with minimum Factor of Safety (FoS) is sometimes not analysed, resulting in over-estimation of FoS of the slope.

This study examines the above issues relating to the design of soil-nailed cut slopes.

2. LIMIT EQUILIBRIUM METHOD OF SLICES

Various methods of slices have been developed for slope stability analysis. The Geotechnical Manual for Slopes (GCO, 1984) recommends the use of non-circular analytical methods, such as those by Janbu (1973) and Morgenstern & Price (1965), for most soil slopes, albeit a sliding block or Bishop (1955) circular analysis may occasionally be more appropriate. At the time of preparation of the Manual, the soil nailing technology was not yet introduced to Hong Kong.

This study reviews and compares the four two-dimensional (2-D) methods of slices that are commonly used in Hong Kong, namely the Janbu's Simplified (JS) method (with and with no correction factor), the Janbu's Generalized (JG) method, the Bishop's Simplified (BS) method and the Morgenstern-Price (M-P) method in respect of soil nail design. These methods mainly differ in the equations of static equilibrium to be satisfied and the relationship between the inter-slice normal and shear forces. Table 1 summarises the conditions of static

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equilibrium satisfied by various limit equilibrium methods of slices. For soil nail design using these methods, it is often assumed that the nail forces do not modify the inter-slice forces. Details of these four methods are discussed in Appendix A.

In addition to the four methods, particular mention is made to the general limit equilibrium (GLE) formulation which was developed by Fredlund in the 1970s (Fredlund & Krahn 1977; Fredlund et al 1981). One special feature of this formulation is that it can produce plots of factor of safety versus λ, where λ defines the relationship between inter-slice shear forces and inter-slice normal forces. The λ plots are useful for understanding the reasons for the differences in the assumptions in the inter-slice shear forces and the conditions of static equilibrium among the JS method, the BS method and the M-P method. They can also be useful to identify the location of FoS. Details of the GLE and the λ plots are presented in Appendix A. The λ plots may also assist in overcoming some non-convergence problems as discussed in Section 7.1.

3. EFFECT OF DISTRIBUTION AND LOCATION OF APPLIED NAIL FORCES

3.1 General

To examine the effect of the location of applied nail forces, stability computations have been performed on a nailed slope and a nailed excavation. The stabilizing effect of soil nails is modelled as external forces on the slope.

To provide a set of nail forces as input in the methods of slices, reference is made to the nail forces distribution derived from analyses using the two-dimensional finite difference code, Fast Lagrangian Analysis of Continua (FLAC). The use of FLAC has little bearing on the present study and hence details are not given in this report.

In the M-P method, the inter-slice function f(x) is assumed to be constant (see Appendix A for details of f(x)). Result of a sensitivity analysis has shown that the constant function yields FoS values almost the same as the half-sine function.

3.2 Unreinforced Model Slope

Figure 1 shows the geometry of the model slope and the material parameters used for the present study. The slope is 20 m in height and standing at an angle of 55°. It has an up-slope angle of 10°. The shear strength parameters of the soil are assumed to be c' = 10 kPa and ϕ' = 43°. FoS values are determined by the four methods of slices. The computer software package SLOPE/W 2004 developed by GEO-Slope was used in the analysis. Ten non-circular slip surfaces (S1 to S10) through the unreinforced slope were considered. Results of the analysis are shown in Figure 2 and the minimum FoS values summarised in Table 2.

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3.3 Model Nailed Slope

3.3.1 Distribution of Axial Nail Forces

Seven rows of soil nails are provided to the model nailed slope (Figure 3) and this corresponds to a vertical nail spacing of 2.5 m. The horizontal spacing of the nails is taken to be 1.5 m. The nails are inclined at an angle 10° below the horizontal. Each soil nail is 20 m long with a 40 mm diameter steel bar in a 100 mm diameter grouted hole.

From the results of FLAC analysis, the distribution of the axial force developed in each of the seven rows of nails at limit equilibrium is given below :

Soil Nail Number	Axial Nail Force, T (kN/m)
SN1	177
SN2	181
SN3	159
SN4	149
SN5	121
SN6	97
SN7	98
	ΣT = 982

Figure 4 shows the locations of nail forces and slip surfaces for the nailed slope model used in the limit equilibrium methods of slices.

3.3.2 Loading Conditions

To investigate the effects of the distribution and location of nail forces, five loading conditions (Figure 5) are considered in the analysis:

(i) Different axial nail forces (T) are applied at individual nail locations;

(ii) The total of the axial nail forces (ΣT) is distributed equally among the seven rows of nails;

(iii) The single nail force (ΣT) is applied near the slope crest, i.e. at location of nail SN7;

(iv) The single nail force (ΣT) is applied at the mid-height of the slope, i.e. at location of nail SN4;

(v) The single nail force (ΣT) is applied near the slope toe, i.e. at location of nail SN2.

In the five loading conditions, the resultant nail forces are the same, i.e. 982 kN/m (ΣT). The nail forces are applied on the slope surface and in the same inclination as the soil nails. The effect of nail force inclination in stability analysis is discussed in Section 5 below.

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3.3.3 Result of Analysis

3.3.3.1 Loading Conditions (i): Different Forces (T) Applied at Individual Nail Locations, and (ii): Total Force Distributed Equally among All Nails

The JG method is unable to produce solutions for any of the slips for the two loading conditions (i) and (ii) due to numerical problems. The problem of convergence encountered in this method is discussed in Section 7.1. The factors of safety computed by using the other methods are plotted in Figure 6. It can be observed that none of the methods can produce a converged solution for slip surface no. S1. Apart from shallow slips (e.g. S2 and S3), the BS, JS and M-P give similar FoS values at given slips for both loading conditions. The difference in FoS for shallow slips can be up to 14%, whereas that for deeper slips is smaller than 5%.

Figure 7 shows the FoS versus λ plots for slip surface S8 for loading condition (i), and loading condition (ii). This illustrates the small difference in computed FoS using the different methods.

3.3.3.2 Loading Condition (iii): Single Force (ΣT) Applied at Top of Slope

Again, the JG method is unable to produce solutions for any of the slips due to numerical problems. Figure 8 shows the FoS obtained for the other three methods at different slip surfaces. The BS method produces converged solutions for all the ten slip surfaces. The JS and the M-P methods cannot compute converged solutions for some of the slip surfaces. The slip surface with the minimum FoS cannot be located for the M-P method due to numerical non-convergence for slips S1 to S5. However, the trend of the FoS curve shows that the minimum FoS is close to that of S6. For a given slip, the JS method gives the highest FoS and this is followed by the M-P method. The BS method usually gives the lowest FoS. The differences between the Bishop FoS and the Morgenstern-Price FoS are however small.

For illustrative purpose, the FoS versus λ plot for slip S8 is presented in Figure 7.

3.3.3.3 Loading Condition (iv): Single Force (ΣT) Applied at Mid-slope Height

The JG method is unable to produce solutions for any of the slips due to numerical problems. Figure 9 shows the FoS obtained for the other three methods at different slip surfaces. All the methods give similar values for the FoS. Both the M-P and the JS methods have convergence problems for the shallow and steep slips surfaces, S1 to S3. The BS method cannot give a converged solution for slip S1 only.

For illustrative purpose, the FoS versus λ plot for slip S8 is presented in Figure 7.

3.3.3.4 Loading Condition (v): Single Force (ΣT) Applied near Slope Toe

The JG method is unable to produce solutions for any of the slips due to numerical problems. Figure 10 shows the FoS obtained for the other three methods at different slip

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surfaces. All the three methods have problem of convergence for some of the shallow slips.

For illustrative purpose, the FoS versus λ plot for slip S8 is presented in Figure 7.

3.3.3.5 <u>Single Nail Force Applied at different Locations</u>

The JG method did not yield any solution for any of the slips because of convergence problem. As such, no comparison of this method can be made.

Figure 11 shows the factor of safety values computed using the JS method for the three cases of a single nail force applied at different locations, i.e. loading conditions (iii), (iv) and (v) respectively. The computed FoS are very similar for the three applied locations. This shows that the point of application of nail force has insignificant effect on the result. This is because the JS method does not consider moment equilibrium, and hence the point of application of external force, which affects the lever arm for moment computation, does not come into play.

The factors of safety computed using the BS method for the three loading conditions (iii), (iv) and (v) are plotted in Figure 12. For those converged slip surfaces, loading condition (v) gives the highest FoS, whereas loading condition (iii) gives the lowest FoS.

Figure 13 shows the results for the M-P method. A similar finding of increasing computed FoS from loading condition (iii) to (v) is seen. That means applying the nail force at the slope toe gives a higher FoS than applying at the higher part of the slope. The reason for this is that the location of applied force controls the magnitude of resisting moment in the moment equilibrium methods (see equation A2 and Figure A1 in Appendix A). Lower the location of the applied nail force is, larger the moment arm and the resisting moment are.

3.3.3.6 <u>Single Nail Force versus Distributed Nail Forces</u>

The minimum FoS computed using different methods are shown in Figure 14 and summarized in Table 3 for the five loading conditions.

Before application of soil nails, all five methods show that the slip surface with minimum FoS is shallow and at S2 as shown in Table 2. With the presence of soil nails, the slip surfaces with minimum FoS become more deep-seated. Their locations vary slightly from one to another among the five methods and also the loading conditions (see Table 3). In practice, the slip surface with minimum FoS is not necessarily the most critical one for the determination of soil nail forces.

There is a narrow disparity in the minimum FoS computed using the JS method amongst the five loading conditions. This means that as long as the total nail force is the same, the assumption of distributed forces or a single force does not affect much the results of the JS method.

For the BS and M-P methods, there are only small differences in the computed minimum FoS for loading condition (i), (ii) and (iv). This can be explained. In each of

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these loading conditions, the resultant nail force is applied either at or close to the mid-height of the slope. Therefore, the magnitude of the resisting moments due to the nail forces are similar. In fact the resultants of different systems of nail forces having the same magnitude and the same line of action on the slope should produce the same FoS.

The above can be looked at further by examining the FoS versus λ plots. Figure 7 depicts such plots for slip S8 for the five loading conditions. For ease of illustration, they are combined into one plot in Figure 15. This shows that the FoS for moment equilibrium is sensitive to the location of the resultant nail force because nail forces applied at different locations will have different moment arms.

3.4 Nailed Excavation

Stability computations similar to those of the nailed slope have also been carried out on a nailed excavation. The details and soil parameters used are shown in Figure 16. The excavation is 6m deep and is supported by four rows of 8m long soil nails installed at 10° below the horizontal. Details of the staged excavation are given in Appendix B.

Eleven slip surfaces (S1 to S11) through the top and toe of the excavation are considered in the analysis, see Figure 17. Similar to the nailed slope presented above, five loading conditions are considered:

(a) Different axial nail forces (T) are applied at individual nail locations;

(b) The total of the axial nail forces (ΣT) is distributed equally among the four rows of nails;

(c) The single nail force (ΣT) is applied near the top of excavation, i.e. at location of nail N4;

(d) The single force (ΣT) is applied at the mid-height of the excavation;

(e) The single force (ΣT) is applied near the bottom of the excavation, i.e. at location of nail N1.

Results of the stability computations using the various limit equilibrium methods are presented and discussed in Appendix B. In general, the results are similar to those obtained for the nailed slope except that there are numerical problems in locating slips with minimum FoS in most of the computations for the nailed excavation.

3.5 Discussion

Wan & Yue (2004) also reviewed different limit equilibrium methods for soil-nailed slope design. Their study considered 16 typical soil-nailed slopes covering a range of effective slope heights from less than 7.5 m to greater than 30.0 m. The M-P and the JS

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methods were applied on the same set of slip surfaces in each design case. The results brought to the conclusion that in all cases before stabilization, the JS method (with no correction factor) computed up to 10% lower minimum FoS values than those given by the M-P method, implying a more conservative design when using the JS method (with no correction factor).

The present study shows that the observation made by Wan & Yue (2004) is not always the case but depends on the problem setting. As indicated in Figure 14, under loading condition (iii), the minimum FoS given by the JS method (with no correction factor) is higher than that given by the M-P method. This is because the JS method only satisfies force equilibrium and is insensitive to the location of applied force. In the case like loading condition (iii), the solutions given by the JS method (with no correction factor) will be on the unsafe side. The JS method, whether a correction factor is applied or not, gives an approximate solution, which will be good enough for analysis that does not involve an externally applied force. In the case of soil-nailed slope, the error associated with the JS method due to ignoring the moment equilibrium may either err on the conservative or on the unsafe side, depending on the locations of the nail forces.

The M-P method satisfies both force and moment equilibrium of slices and gives a more exact solution. The present study confirms that the method is sensitive to the assumption on the locations of nail forces. The result will be true only if the locations of nail forces are correctly assumed. The lines of action of forces should correspond to the locations of soil nails.

4. EFFECTS OF NAIL LOADS APPLIED AT SLOPE FACE AND AT SLIP SURFACE

4.1 General

In the previous analysis, the nail forces are applied on the face of slope or excavation. There is always a question as to whether the nail forces should be applied at locations where the line of action of the nail forces intersects the slip surface. This Section compares the effect of applying the nail forces at the slope/excavation face with that of applying the nail forces at the slip surface. The comparisons make use of the models of nailed slope and nailed excavation described in Sections 3.3 and 3.4 respectively. The scenario of applying different nail forces is used in this comparative analysis, i.e. loading condition (i) for the nailed slope and loading condition (a) for the nailed excavation.

4.2 Nailed Slope

For the case when the nail forces are applied on the slope face, the factors of safety computed by using the various methods for slip surface S8 are summarised in Table 4. Figure 18 shows the nail forces and the slices. For illustrative purpose, free body diagram and force polygon showing the inter-slice forces for slice no. 10 are presented in Figure 19; the middle nail force (SN4) is applied at this slice. The inter-slice normal forces on the two sides of the slice are in compression.

For the case when the nail forces are applied at where their lines of action intersect the slip surface S8, Figure 20 shows the nail forces and the slices. Free body diagram and force

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polygon showing the inter-slice forces for slice no. 20 with the nail force of SN 4 are shown in Figure 21. The inter-slice normal forces on the two sides of the slice are in compression.

It can be noted in Table 4 that the factor of safety values in these two cases are very similar. The maximum difference is 4.6% less in the latter case when using the JS method (with correction factor).

4.3 Nailed Excavation

For the case the nail forces are applied at the excavation face, the factors of safety computed by using the various methods for slip surface S7 are summarised in Table 5. The nail forces act on the vertical face of the excavation at slice 30 (see Figure 22):

For illustrative purpose, free body diagram and force polygon for slice 12 are presented in Figure 23. The inter-slice normal forces on the two sides of the slice are compression. Figure 24 shows the distribution of the inter-slice shear and normal forces. The normal force increases evenly and gradually, and drops sharply to zero at the last slice (slice 30). The direction of the inter-slice shear force is reverse of that which usually occurs when only the self-weight of the slice is considered. The shear stress reversal is an indication of negative λ.

For the case where the nail forces are applied at where the line of action of the nail forces intersects the slip surface, the nail forces and slices are shown in Figure 25. The FoS calculated by the different methods of slices are summarised in Table 5. It can be noted that the two methods of load application produce nearly the same values of factor of safety. The maximum difference is about 4% larger in the latter case when using the BS method.

Free body diagram and force polygon for slice no. 19 with the nail force of N2 are shown in Figure 26. The force polygon closes, indicating that the slice is in force equilibrium. However, it should be noted that the inter-slice normal force on the right hand side of the slice is in tension. Indeed, many other slices also have tension forces, as indicated in the inter-slice force plot in Figure 27. These tensile inter-slice normal forces appear to be unreasonable. Despite this, all the slices are in complete force equilibrium as indicated by the closure of the force polygons for each slice. Krahn (2003) opined that once all the mobilised driving forces and base resisting shear forces are integrated, the local irregularities are smoothed out, making the overall factor of safety for the entire sliding mass acceptable.

4.4 Discussion

The analytical results for both the nailed slope and the nailed excavation illustrate that the application of loads on the slope face and that on the slip surface produce very similar values of factor of safety. In fact different distribution of nail forces having the same magnitude and the same line of action of resultant force should produce similar FoS. Nevertheless, in the nailed excavation analysis, applying nail forces on the slip surface gives unreasonable tensile inter-slice normal forces.

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5. EFFECT OF USING HORIZONTAL COMPONENT OF NAIL FORCE AND USING NAIL FORCE ALONG THE SAME INCLINATION OF SOIL NAIL

5.1 General

In reinforced slope construction, reinforcements are generally placed horizontally in compacted fill. The tension force provided by the reinforcement acts in a horizontal direction. In the event that the reinforcements are inclined to the horizontal, only the horizontal component of tension force is considered effective in maintaining slope stability. This concept was borrowed for soil-nailed slope design in the early 1990s.

Soil nails are always installed at an angle to the horizontal. Where the inclination of soil nails to the horizontal is small, neglecting the vertical component of the nail force should not give a much different solution. The difference gets larger when the inclination is increased. In current practice, some designers apply nail force in the same direction as the nail alignment in stability analysis while some just consider the horizontal component of the nail force.

5.2 Analysis

Stability analysis has been carried out to compare design using horizontal components of nail forces to that using the nail forces along the same inclination of soil nails. The Morgenstern-Price method is used in the stability analysis.

The slope model used in the analysis is 20 m high with an angle of 60°. It has an upslope angle of 20°. The slope is reinforced with three layers of soil nails. For the purpose of stability analysis, each nail is assumed to provide an axial force of 200 kN/m, which acts on the slope surface in the same inclination of the nail (ω). The relatively large magnitudes of soil nail forces are so chosen that the inclination effect of soil nails in the analysis may be seen more easily in this illustrative example. The geometry of the slope, locations of the soil nails and the assumed soil strength parameters are shown in Figure 28.

Nine slip surfaces (1 to 9) through the toe of the slope have been considered. In the analysis the inclination (ω) of the nail forces is varied between 5^o and 30^o below the horizontal, at an interval of 5^o. For each inclination, the factors of safety of the slip surfaces are determined using two approaches: (a) nail forces are applied in the same inclination of the nails; and (b) horizontal forces, which are the same as the horizontal component of the inclined forces, are applied to the slope.

5.3 Results of Analysis

The results of the comparative analysis are summarized in Table 6. It can be seen from the last column of the Table that the differences in the computed factors of safety between the two approaches generally increase as the nail inclination increases. For very shallow slip surface, the factor of safety computed using the horizontal component forces is greater than that using the inclined forces. The difference obtained from the present analysis is up to about 11%. The factor of safety computed using the horizontal components becomes closer to and eventually smaller than that computed using the inclined forces when

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the slip surface goes deeper. The difference for the deepest slip (slip surface no. 9) is not more than 2.4%. For the slip surface with the minimum FoS, the maximum difference is only 4.9%, when the inclination angle (ω) of the nail is 30°.

The approach of using the horizontal force component essentially ignores the vertical component of the nail force. The magnitude of the vertical force component and its effect increases with increase in the nail inclination.

A simple hand calculation has been carried out to examine the contribution of shearing resistance by the horizontal and the vertical components of nail forces at various nail inclinations and slip surface orientations. The results are given in Appendix C. Depending on the angle between the nail and the normal to the slip surface (θ), the vertical component of the nail force could have a positive or a negative contribution to the shearing resistance along the slip surface. For very steep slip surface (e.g. shallow slip) where θ is small, the vertical component of the nail force generally has a negative contribution. Hence ignoring the vertical component will result in a higher computed factor of safety. The effect of the vertical component is not significant for small nail inclinations but increases with increasing nail inclination. These results confirm the findings of the comparative analysis.

As soil nails are normally installed at small inclinations to the horizontal, the differences in factor of safety computed using the two approaches are generally small. Nevertheless, it is preferable to apply the forces in the direction of the soil nails in order to obtain a more realistic model. This approach is used in overseas design codes such as Phear et al (2005), Department of Transport (1994), French National Research Project (1991), FHWA (1998) and JHPC (1998).

6. METHODS FOR SEARCHING SLIP SURFACE WITH MINIMUM FACTOR OF SAFETY

It is important in slope stability analysis for geotechnical design that the slip surface with the minimum factor of safety is identified. When circular slip surfaces are considered, the slip surface with the minimum FoS can be searched automatically by repeated trials of a number of slip surfaces. One conventional method employed in many computer programmes is to specify a grid pattern of centres for a family of circular slip surfaces, and then locate the slip with the minimum computed factor of safety. For non-circular slip, it is much more difficult to automate the search. Many computer programmes require the user to locate such a slip surface by analysing a number of admissible trial slip surfaces. Judgements are needed when determining the slips to be analysed. The following discussions apply to scenarios both before and after the application of soil nailing.

From mathematical point of view, the location of slip surface with minimum factor of safety is an optimization process. It involves formulation of an objective function (e.g. factor of safety), which consists of some prescribed variables (e.g. location of slip) with a number of constraints. By varying the prescribed variables, an optimum (e.g. minimum) value of the objective function can be obtained. There are many mathematical techniques available for the optimization process. Some of these techniques have been employed in geotechnical engineering application. For examples, Nguyen (1985) proposed the "simplex reflection technique", and Chen & Shao (1988) used the "simplex, steepest descent and

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Davidson-Fletcher-Powell method" for locating such slip surface. Locally, Cheng (2003) has developed a computer programme called SLOPE 2000 that adopts the simulated annealing technique to locate the slip surface with the minimum FoS. The concept of the simulated annealing technique is presented in Appendix D.

For the computer software SLOPE/W 2004, an optimization process of incrementally changing only portions of a slip surface has been introduced to search for the non-circular slip surface with the minimum FoS. Details of the method can be found in Krahn (2004).

There are other techniques for searching the slip surface with the minimum FoS. Examples are the "random slip surface generators" (Boutrop & Lovell, 1980; Siegel et al., 1981), the "alternating-variable" technique (Celestino & Duncan, 1981), the "programming technique" (Arai & Tagyo, 1985), and the "spread sheet" technique (Low & Tang, 1997).

One limitation of automatic search of the slip surface with the minimum FoS is that it does not consider the scale of failure. A slip surface with the minimum factor of safety may involve only a small volume of soil at localized area near the slope surface, especially when the cohesion (c') value is small. The consequence of failure associated with such a slip is small. Yet, slip surfaces with slightly higher factors of safety may involve much larger soil volume with more significant consequence in the event of failure.

Although the mathematical optimization techniques are often employed to determine the slip surface with the minimum factor of safety, they should be used with care and judgement. It is because the slip surface derived from such techniques may not always be realistic because of simplification in the ground model by the designer for the stability analysis. Sensitivity analyses have been carried out on unreinforced and nailed slopes using the common range of shear strength parameters for Hong Kong soils. The results show that SLOPE/W 2004 and SLOPE 2000 sometimes produce minimum FoS slip surfaces that pass below the toes of the slopes. As an example, Figure 29 shows the result of an automatic search for the slip surface with the minimum FoS for loading condition (ii) for the nailed slope, using the SLOPE/W 2004 and SLOPE 2000 programmes. Both programmes produce similar results. This reflects the capability of the programmes to locate the slip surfaces with the minimum FoS. In this particular case the minimum FoS slip surface passes below the slope toe. Whether this is credible depends on whether the simplified ground model in actual design cases is good enough or not. For instance, shear strength of the ground material generally increases with depth. This is seldom modelled. It illustrates the importance of detailed and not over-generalized geological models and the possible need to seek engineering geological input when choosing credible slip surfaces for analysis.

7. PROBLEMS AND LIMITATIONS OF METHODS OF SLICES

7.1 Problems of Convergence

A common problem when analyzing the nailed slope and nailed excavation is the difficulty of convergence. This problem is particularly pronounced in the analysis of the nailed excavation in the present study, in which slips of minimum factors of safety cannot be located in most of the stability computations. Details can be found in Appendix B. The situation for the problem is largely associated with (a) the steepness of the slip surface particularly near the crest, and (b) the application of concentrated line loads (nail forces) in

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the upper portions of the slope and excavation. Ching & Fredlund (1983) found that steep slips can cause conditions that give unreasonable m_α values (e.g. zero or negative m_α values) in the numerical procedure and result in convergence problem. The parameter m_α is defined in Appendix A.

The convergence problems are particularly serious for Janbu's generalised method. This is mainly due to the assumption used in the method in respect of the distribution of stress on each slice. The line of thrust is often assumed to be located at one-third of the inter-slice height above the slip surface. The magnitudes of the inter-slice force are determined by taking moments about the base of each slice. As pointed out by Krahn (2003), this approach generally works well when the potential sliding mass has no significant stress concentrations. If stress concentrations exist that deviate significantly from the assumed stress distribution, convergence problems can occur. This is particularly true when features like anchors or nails are included in the analysis.

The developer of the computer software SLOPE/W 2004 (Krahn, 2004) suggested that the convergence problem could be resolved by relaxing the convergence tolerance and allowing the review of an additional grid point to see if the factors of safety changes or whether the values are increasing or decreasing.

In SLOPE/W 2004, the lambda (λ) plot is obtained by calculating the FoS within the default range of λ from -1.25 to +1.25 for both the force equilibrium and the moment equilibrium equations. The intersecting point represents the λ value and the FoS for the M-P method where both the force and the moment equilibrium are satisfied. The default range of λ is so set because an intersecting point can be found within this range in most of the cases. However in some cases, even though an intersecting point is within the default range of λ, the computation may stop and no FoS can be found. It is because during the computation process when λ approaches -1.25 or +1.25 (i.e. when the inter-slice forces are steeply inclined to the horizontal), a non-convergence problem may be encountered. In these cases Krahn (2004) suggested that the user can manually re-set λ to a narrower range to avoid the problem of non-convergence.

Cheng & Lansivaara (2005) reported the development of a software programme called "SLOPE 2000" for slope stability analysis. The programme makes use of a numerical approach called the "double QR method" to get close form solutions. This helps reduce the non-convergence problem.

7.2 Limitations of Limit Equilibrium Method of Slices

The limit equilibrium methods of slices have been widely and successfully used. However, according to Wright et al (1973), these methods have several fundamental shortcomings. These shortcomings are presented and discussed in Appendix E.

Behaviour of nailed structures is also a strain compatibility problem. A nail force develops through the interaction among the deforming soil, the soil nail and the nail head. Shiu & Chang (2006) carried out numerical analyses on soil slopes installed with steeply inclined nails. They reported that the reinforcing action of soil nails can be reduced significantly with an increasing nail inclination (ω) (Figure 30). Depending on the

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orientation angle (θ) made between the nail and the normal to the slip plane, compressive forces rather than tensile forces can be mobilised in soil nails. For steeply inclined nails, the nail forces change from tension to compression when θ changes from positive to some negative values. This contradicts the common design assumption used in limit equilibrium methods that only tensile nail forces are developed.

In cramped sites, steeply inclined soil nails may have to be used. One method to determine the nail forces for the strain compatibility problem is to use finite element or the finite difference method. Alternatively, if the limit equilibrium method is to be used, each soil nail should be placed in a positive orientation angle θ with respect to the normal of the potential slip plane such that the axial tensile nail force will be developed. The optimal value of θ for maximising the mobilized tensile capacity of the soil nail is around 45°. This is for reference only as it is not practicable in design to orient every soil nail to this inclination.

Furthermore, for the limit equilibrium methods, it is possible to define a wide variety of nail length patterns that satisfy stability requirements but that may not be satisfactory in terms of serviceability. It is especially the case for soil-nailed excavations using the top-down method of construction. An example of this can be found in Shiu & Chang (2006).

Although the methods of slices involve assumptions and have certain weaknesses, they do provide a useful and practical technique for the analysis of slopes (both unreinforced and reinforced) and other geotechnical engineering problems. It is important that the user should understand the limitations of these methods. The user should also recognize the potentially erroneous results and interpret the results carefully.

8. CONCLUSIONS AND RECOMMENDATIONS

Stability computations have been performed on a nailed slope and a nailed excavation using various methods of slices including the Bishop's Simplified method, the Janbu's Simplified method, the Janbu's Generalised method and the Morgenstern-Price method. Both the nailed slope and the nailed excavation give similar computational results. This similarity provides additional confidence.

The following main observations are made from the computational results:

(a) The FoS values computed using the Bishop's Simplified method and the Morgenstern-Price method are sensitive to the assumed locations of nail forces.

(b) The FoS values computed using the the Janbu's Simplified method are insensitive to the assumed locations of nail forces. This is an inherent limitation of the method.

(c) If the applied resultant nail force is located above the actual resultant nail force, the Janbu's Simplified method tends to give higher FoS values than those given by the

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Morgenstern-Price method. The Janbu's simplified method may give a conservative or an unsafe solution depending on the nail pattern and locations.

(d) For the four methods reviewed, the FoS values derived based on the actual loading condition (i.e. condition (i) for the nailed slope and condition (a) for the nailed excavation) are close to those based on the two conditions of evenly distributed nail forces and single nail force applied at the mid-height, as the locations of the resultant force of the three loading conditions are similar.

(e) Nail forces applied on the surface of slope or retaining wall give FoS values very similar to those given by the forces applied on the slip surface where the lines of action intersect the slip surface. However, the inter-slice forces in the latter case may not be reasonable (see Section 4.3).

(f) All the methods have encountered convergence problems. The problems are most serious for the Janbu's Generalised method which has not produced converged solutions for any of the slips analysed.

(g) The approach of using the nail force in the same inclination of the soil nail and that of using the horizontal component of the nail force produce different FoS values. The latter ignores the vertical component of the nail force. The difference is not significant for small nail inclinations commonly used in design.

(h) The limit equilibrium methods have some limitations. Such limitations should be borne in mind when interpreting the results of stability computations, especially when the stress-strain assumptions may not be valid (e.g. when compressive forces are mobilized in steeply inclined nails).

The following recommendations are made for the design of soil nails in cut slopes using limit equilibrium methods:

(a) Methods which satisfy both force and moment equilibrium such as the Morgenstern-Price should be used for design.

(b) Where problem of convergence is encountered, the convergence tolerance may be relaxed. This needs to be carried out with good engineering judgment. The accuracy of the limit equilibrium method used should be taken into account. The relaxation should only be applied to the non-converged slip surfaces. Plotting FoS values versus λ values helps identify and understand the convergence

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problem. In some cases the problem may be resolved by changing the default range of λ in SLOPE/W 2004. In addition, other numerical algorithms such as the Double QR method may help solve the non-convergence problem.

(c) If the nail forces on the slope are modelled by using a single force, this force should be located near the expected line of action of the resultant of nail forces.

(d) Nail forces should preferably be applied in the same inclination as the soil nails, though the approach of considering only the horizontal components of the nail forces is considered acceptable.

(e) Each soil nail should be placed in an orientation with respect to the normal of the potential slip plane such that axial tensile force will be developed (see Section 7.2). If in doubt, numerical methods such as the finite element or the finite difference methods may be used to supplement the analysis.

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Table 1 - Elements of Statical Equilibrium Satisfied by Various Limit Equilibrium Methods of Slices

Method	Force Equilibrium		Moment Equilibrium
	Vertical	Horizontal	
Bishop’s Simplified	Yes	No	Yes
Janbu’s Simplified	Yes	Yes	No
Janbu’s Generalized	Yes	Yes	**
Morgenstern-Price	Yes	Yes	Yes

Legend:

** Moment equilibrium is used to calculate interslice shear forces

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Table 2 - Minimum Factor of Safety (FoS) Using Different Methods of Analyses for the 20 m High Model Slope

Methods in Slope/W	
Morgenstern - Price	1.13 (S2)
Bishop’s Simplified	1.17 (S2)
Janbu’s Simplified with Correction Factor	1.13 (S2)
Janbu’s Simplified with No Correction Factor	1.10 (S2)
Janbu Generalized	1.13 (S2)
Notes: ( ) Indicates the slip surface no. with the minimum factor of safety	

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Table 3 - Minimum Factors of Safety Computed Using Different Methods of Analyses

Methods	Loading Condition (i)	Loading Condition (ii)	Loading Condition (iii)	Loading Condition (iv)	Loading Condition (v)
Morgenstern - Price	2.39 (S8)	2.32 (S8)	*1.82 (S6)	2.32 (S7)	2.61 (S8)
Bishop's Simplified	2.43 (S8)	2.33 (S8)	1.69 (S5)	2.33 (S7)	2.76 (S9)
Janbu's Simplified with Correction Factor	2.41 (S8)	2.41 (S8)	2.36 (S9)	2.41 (S8)	2.42 (S8)
Janbu's Simplified with No Correction Factor	2.27 (S9)	2.27 (S9)	2.22 (S9)	2.27 (S9)	2.29 (S8)
Janbu Generalized	NC	NC	NC	NC	NC

Notes: ( ) Indicates the slip surface no. with the minimum factor of safety
NC Non-convergent
* Minimum FoS cannot be searched due to numerical non-convergence for adjacent slip surface

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Table 4 - Summary of FoS Computed by Various Methods for Slip Surface S8 in the Nailed Slope

Method	Slip Surface No.	FoS (Different Nail Forces Applied on Slope Face)	FoS (Different Nail Forces Applied on Slip Surface)
M-P	S8	2.39	2.35
BS		2.43	2.42
JS (with correction factor)		2.41	2.30
JS (with no correction factor)		2.28	2.18

Table 5 - Summary of FoS Computed by Various Methods for Slip Surface S7 in the Nailed Excavation

Method	Slip Surface No.	FoS (Different Nail Forces Applied on Excavation Face)	FoS (Different Nail Forces Applied on Slip Surface)
M-P	S7	1.80	1.77
BS		1.70	1.76
JS (with correction factor)		1.87	1.81
JS (with no correction factor)		1.84	1.87

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Table 6 - Results of Comparative Analysis Using Inclined Nail Force and Horizontal Component of Nail Force (Sheet 1 of 2)

Nail Force in Same Inclination of Soil Nail				Horizontal Component of Nail Force			Difference in FoS (%) (C7-C4)/C4
Inclination, (Degrees) (C1)	Nail Force, F (kN/m) (C2)	Slip Surface No. (C3)	FoS (C4)	Nail Force, F_H (kN/m) (C5)	Slip Surface No. (C6)	FoS (C7)	
5	200	1	NC	199.2	1	NC	-
		2	1.48		2	1.51	2.3
		3	**1.34***		3	**1.36***	1.1
		4	1.36		4	1.37	0.2
		5	1.49		5	1.49	0.1
		6	1.44		6	1.44	-0.1
		7	1.45		7	1.45	-0.2
		8	1.51		8	1.51	-0.3
		9	1.59		9	1.58	-0.4
10	200	1	NC	197.0	1	NC	-
		2	1.43		2	1.50	5.1
		3	**1.32***		3	**1.35***	2.2
		4	1.36		4	1.36	0.4
		5	1.48		5	1.49	0.2
		6	1.44		6	1.43	-0.2
		7	1.45		7	1.45	-0.4
		8	1.52		8	1.51	-0.6
		9	1.59		9	1.58	-0.8
15	200	1	1.78	193.2	1	NC	-
		2	1.39		2	1.48	6.9
		3	**1.30***		3	**1.34***	3.1
		4	1.34		4	1.35	0.6
		5	1.47		5	1.48	0.3
		6	1.43		6	1.43	-0.3
		7	1.45		7	1.44	-0.6
		8	1.52		8	1.50	-0.9
		9	1.59		9	1.58	-1.2

Legend:

NC Non-convergence
* Minimum factor of safety among the nine values

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Table 6 - Results of Comparative Analysis Using Inclined Nail Force and Horizontal Component of Nail Force ((Sheet 2 of 2)

Nail Force in Same Inclination of Soil Nail				Horizontal Component of Nail Force			Difference in FoS (%) (C7-C4)/C4
Inclination, (Degrees) (C1)	Nail Force, F (kN/m) (C2)	Slip Surface No. (C3)	FoS (C4)	Nail Force, F_H (kN/m) (C5)	Slip Surface No. (C6)	FoS (C7)	
20	200	1	1.66	187.9	1	NC	
		2	1.34		2	1.45	8.6
		3	**1.27***		3	**1.32***	3.9
		4	1.33		4	1.34	0.7
		5	1.46		5	1.47	0.3
		6	1.42		6	1.41	-0.4
		7	1.44		7	1.43	-0.8
		8	1.51		8	1.49	-1.2
		9	1.59		9	1.57	-1.6
25	200	1	1.55	181.3	1	NC	-
		2	1.29		2	1.42	10.0
		3	**1.24***		3	**1.30***	4.5
		4	1.31		4	1.32	0.7
		5	1.44		5	1.45	0.4
		6	1.41		6	1.40	-0.5
		7	1.44		7	1.42	-1.0
		8	1.51		8	1.49	-1.4
		9	1.59		9	1.56	-2.0
30	200	1	1.44	173.5	1	NC	-
		2	1.24		2	1.38	11.1
		3	**1.21***		3	**1.27***	4.9
		4	1.29		4	1.30	0.7
		5	1.43		5	1.43	0.4
		6	1.40		6	1.39	-0.7
		7	1.43		7	1.41	-1.2
		8	1.50		8	1.47	-1.7
		9	1.59		9	1.55	-2.4

Legend:

NC Non-convergence
* Minimum factor of safety among the nine values

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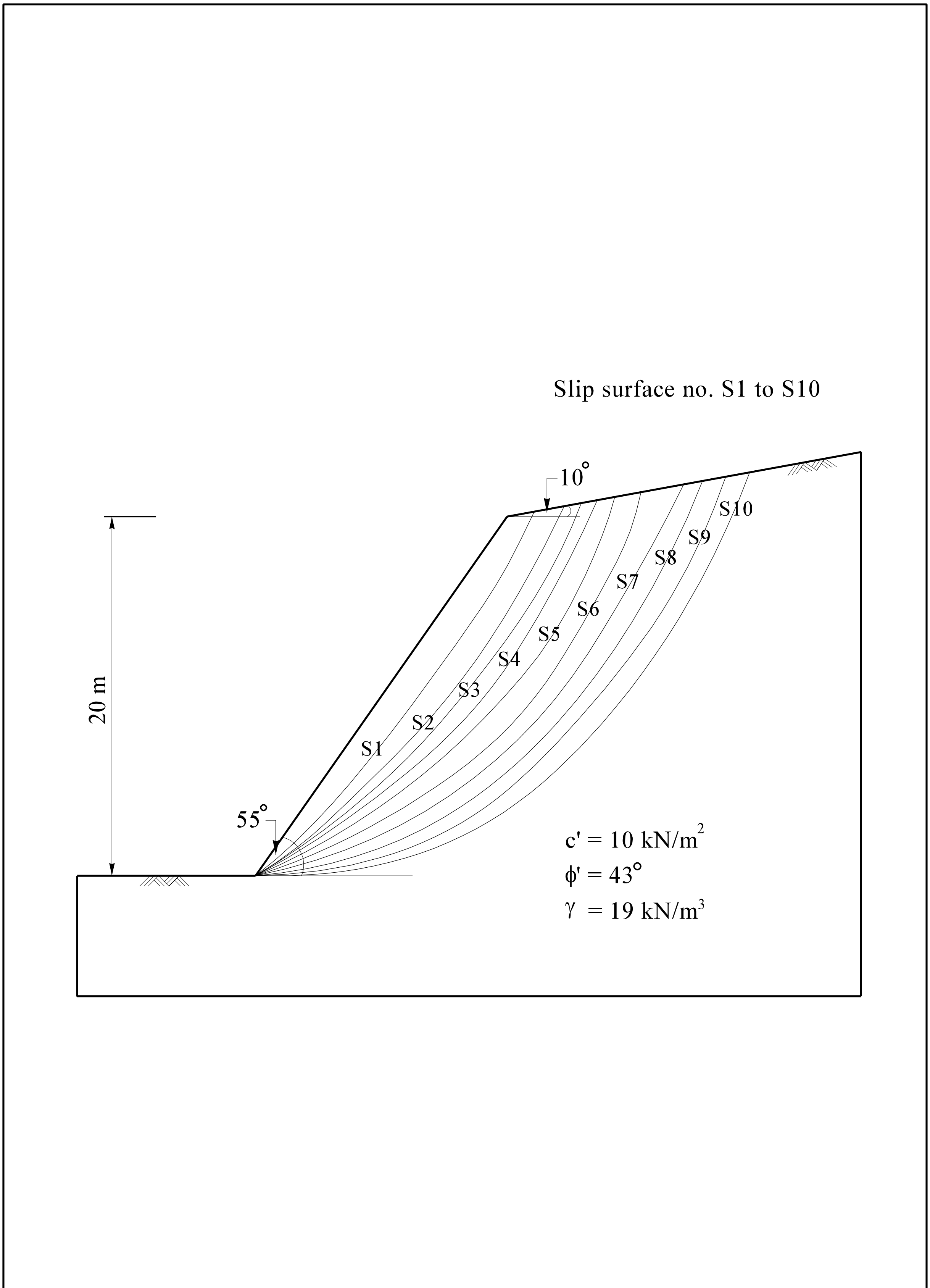

Figure 1 - Geometry and Locations of Slip Surface for the Model Slope

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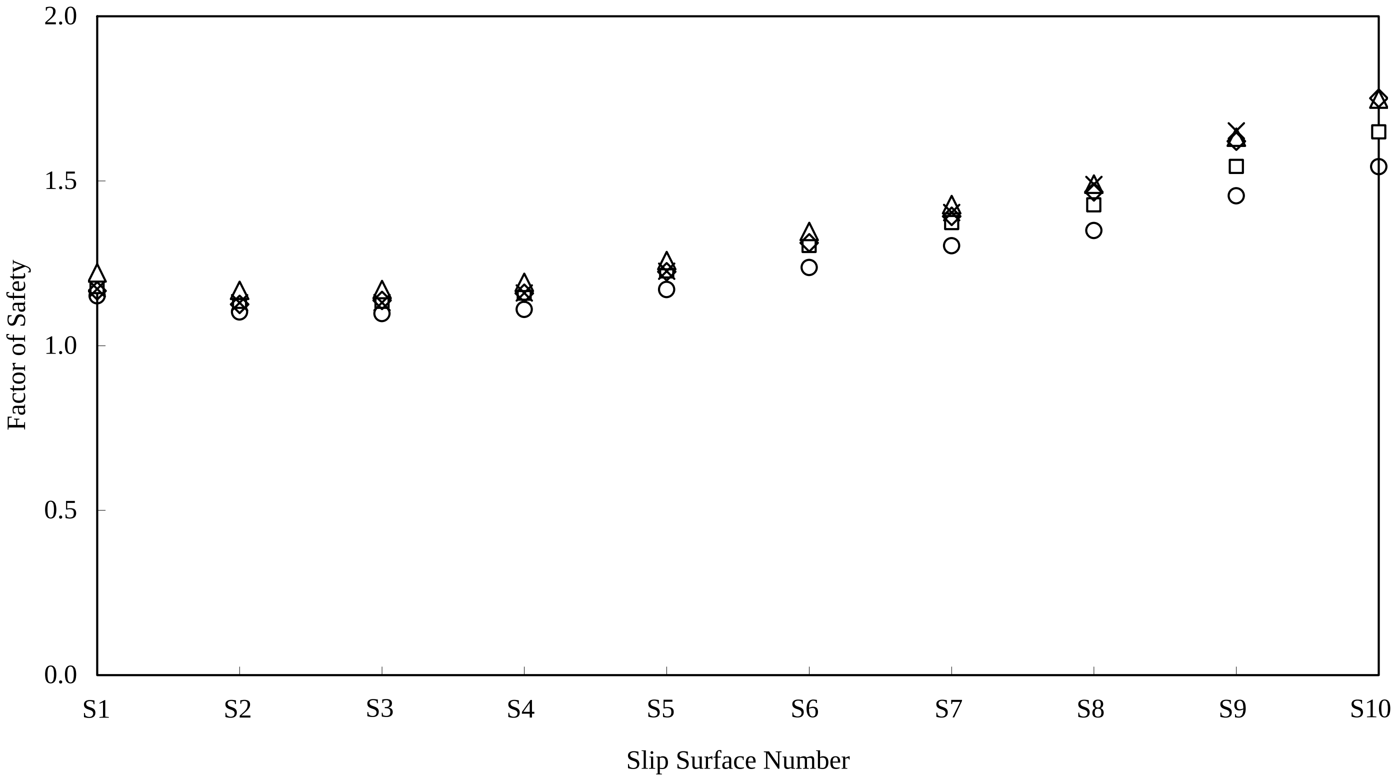

Legend:

◇ Morgenstern - Price

□ Janbu’s Simplified with correction factor

× Janbu’s Generalised

△ Bishop’s Simplified

○ Janbu’s Simplified with no correction factor

Figure 2 - Factors of Safety of the Unreinforced Slope Using Different Limit Equilibrium Methods

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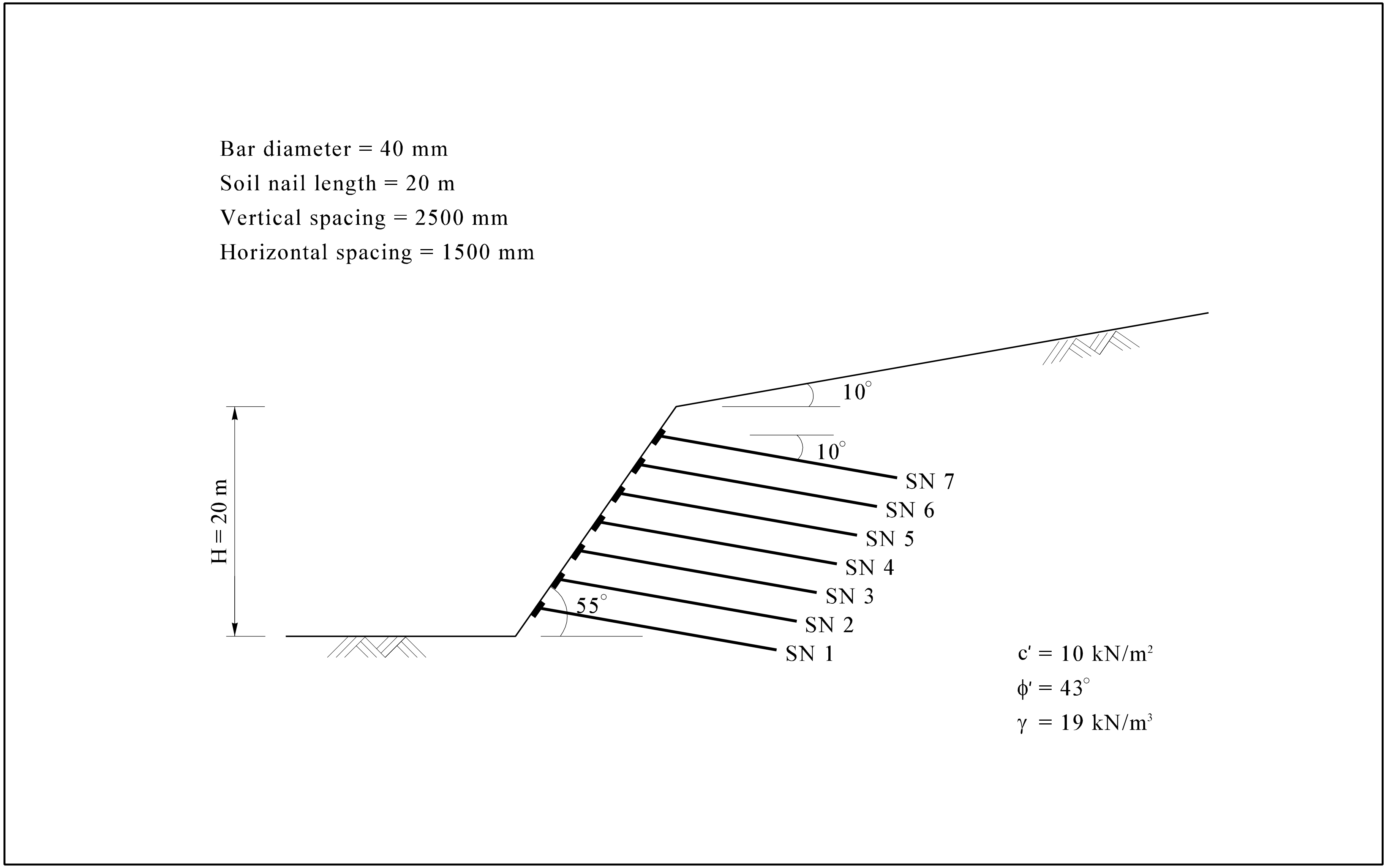

Figure 3 - Geometry and Material Parameters of Model Nailed Slope

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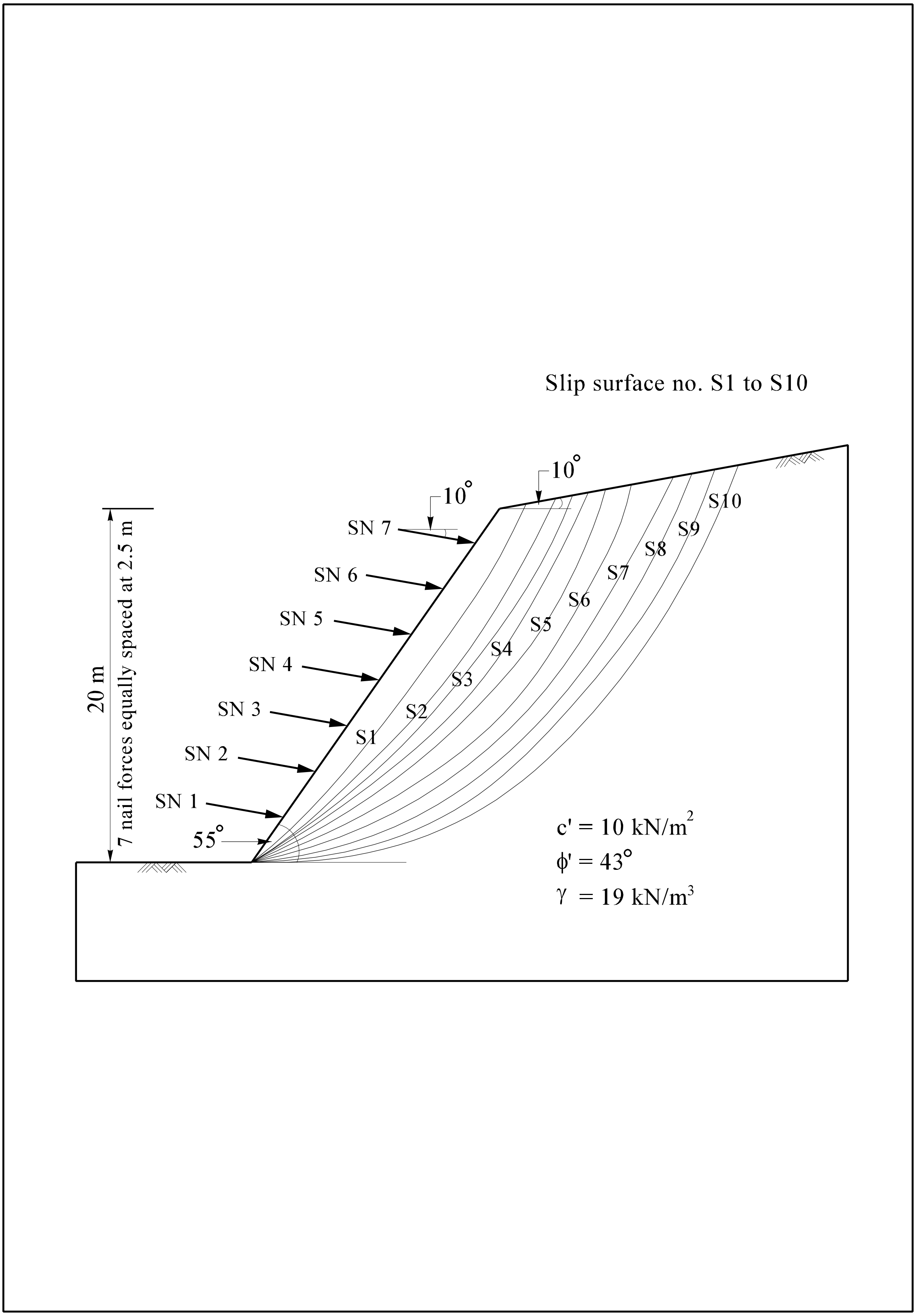

Figure 4 - Locations of Nail Forces and Slip Surfaces for the Model Nailed Slope

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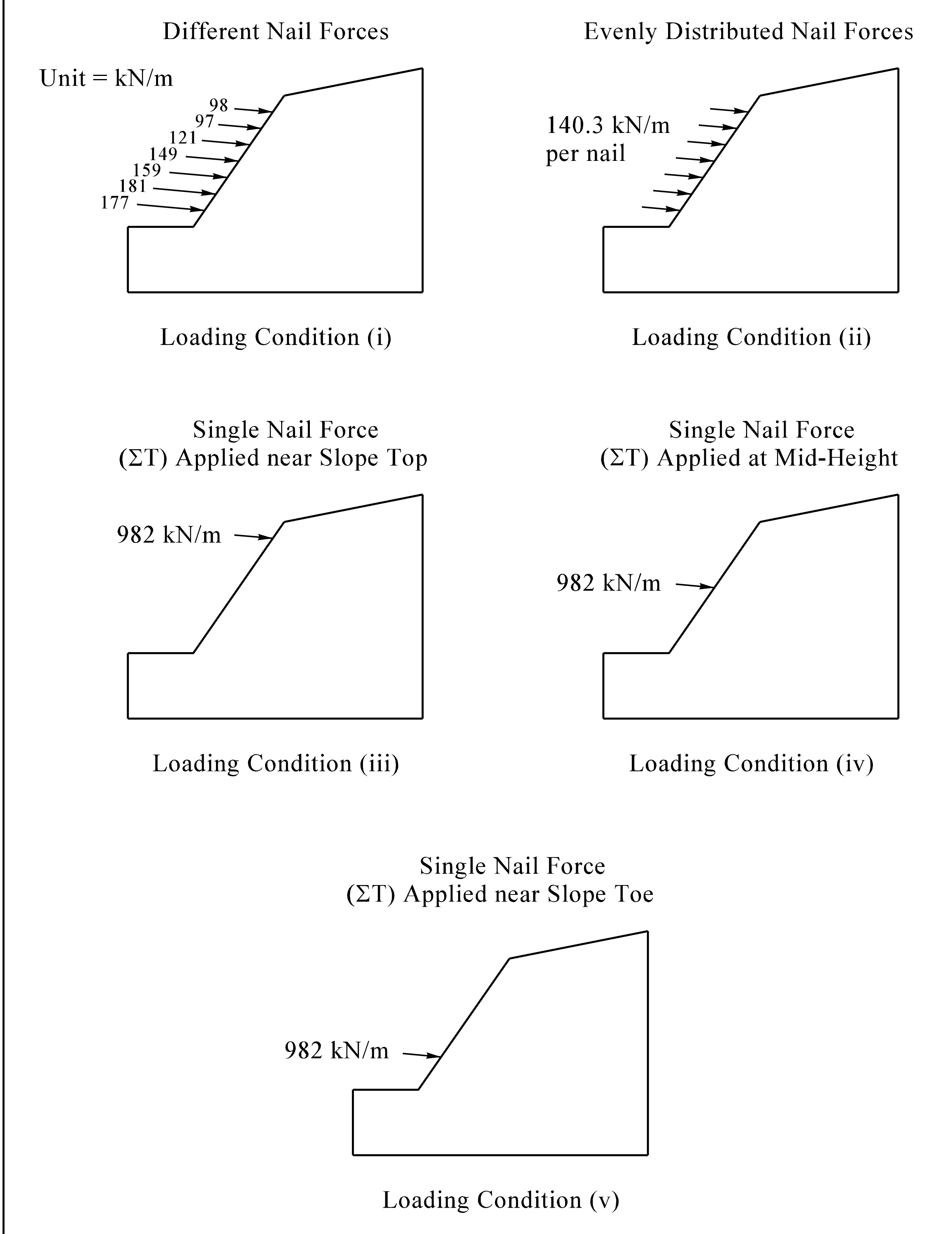

Note: Distribution of nail forces for loading condition (i) is obtained from FLAC analyses.

Figure 5 - Loading Conditions for Nailed Slope

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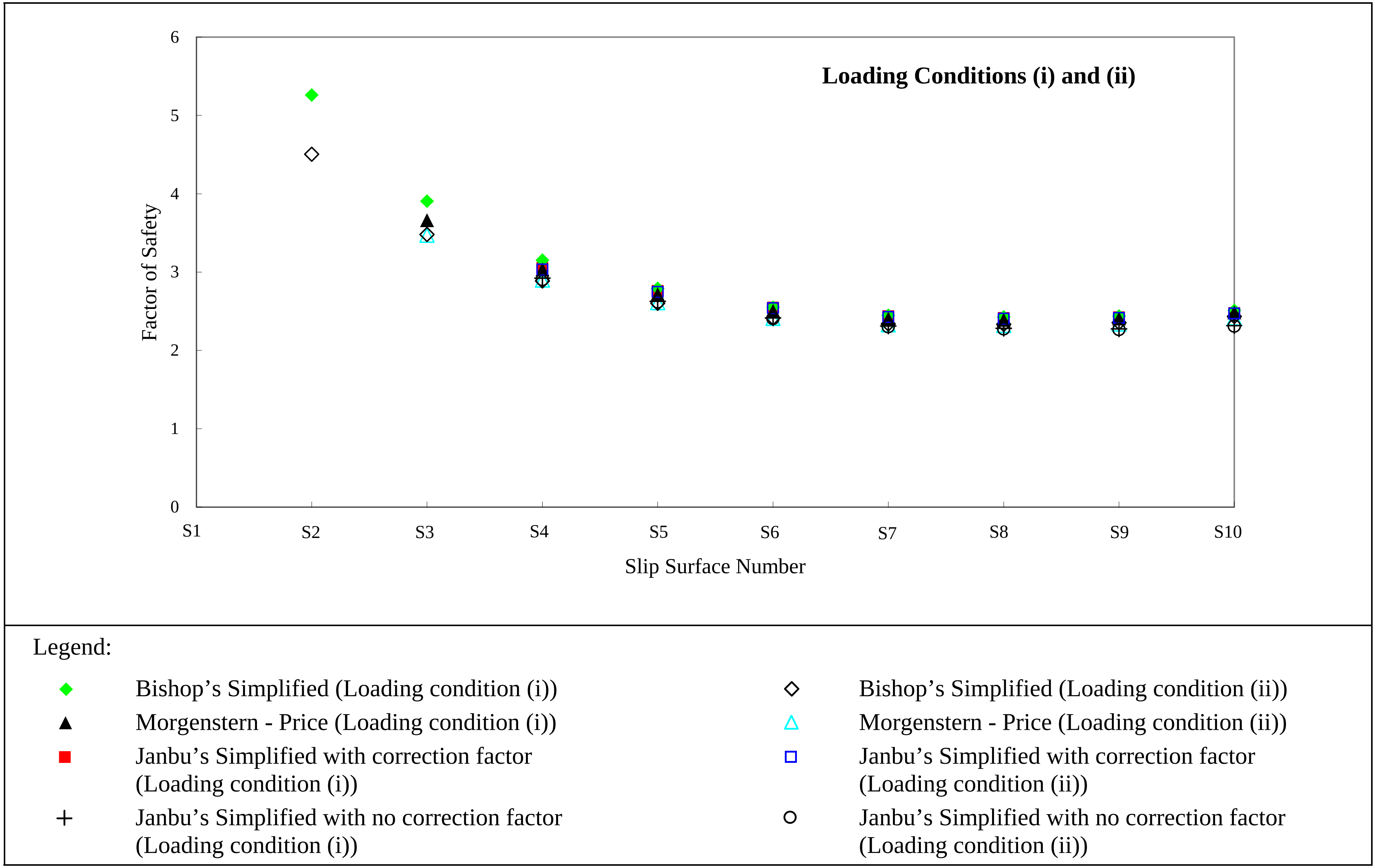

Figure 6 - Factor of Safety versus Slip Surface Number for Loading Conditions (i) and (ii)

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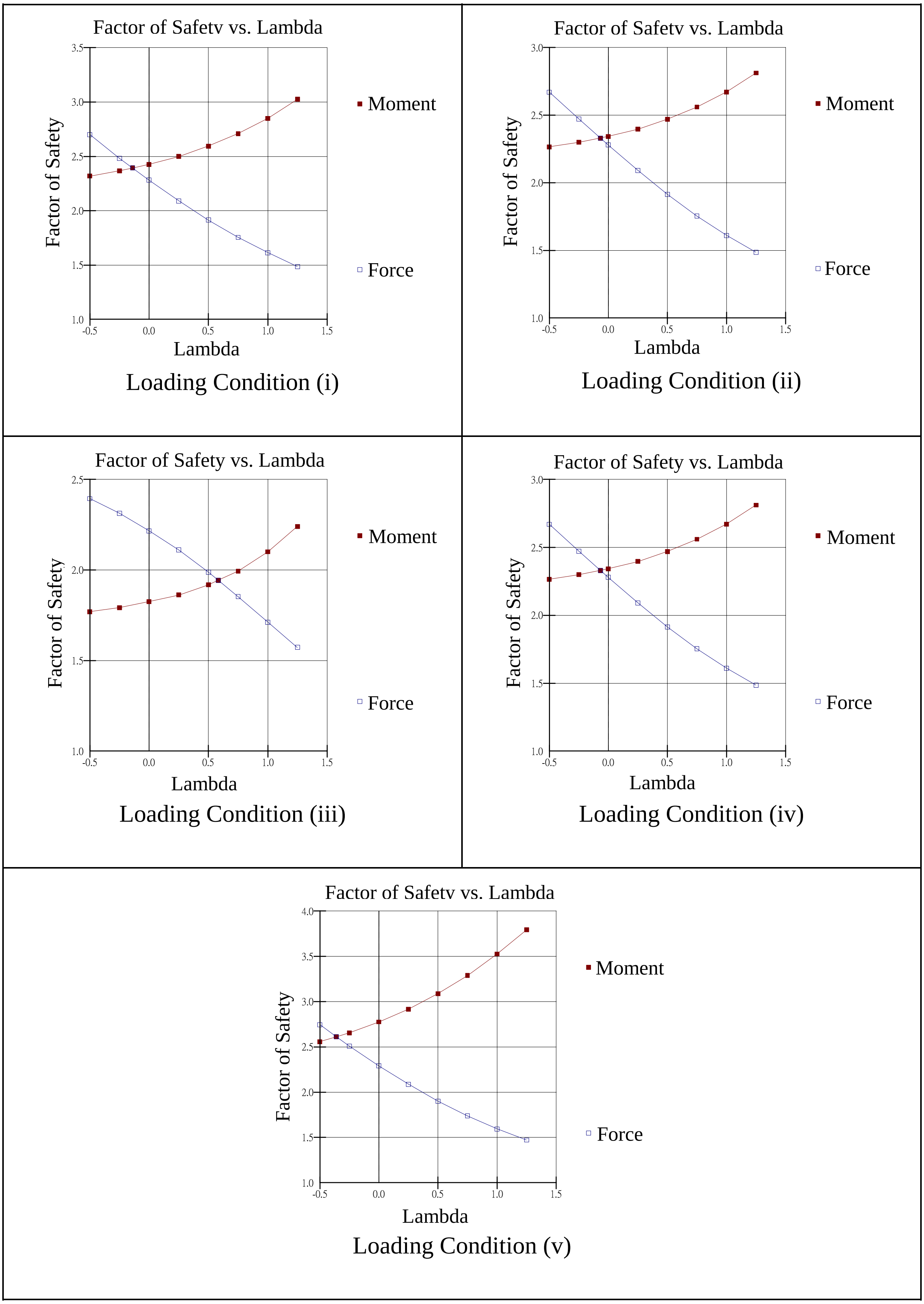

Figure 7 - Factor of Safety (FoS) versus Lambda (λ) for Slip Surface S8 for Loading Conditions (i) to (v) in the Nailed Slope Model

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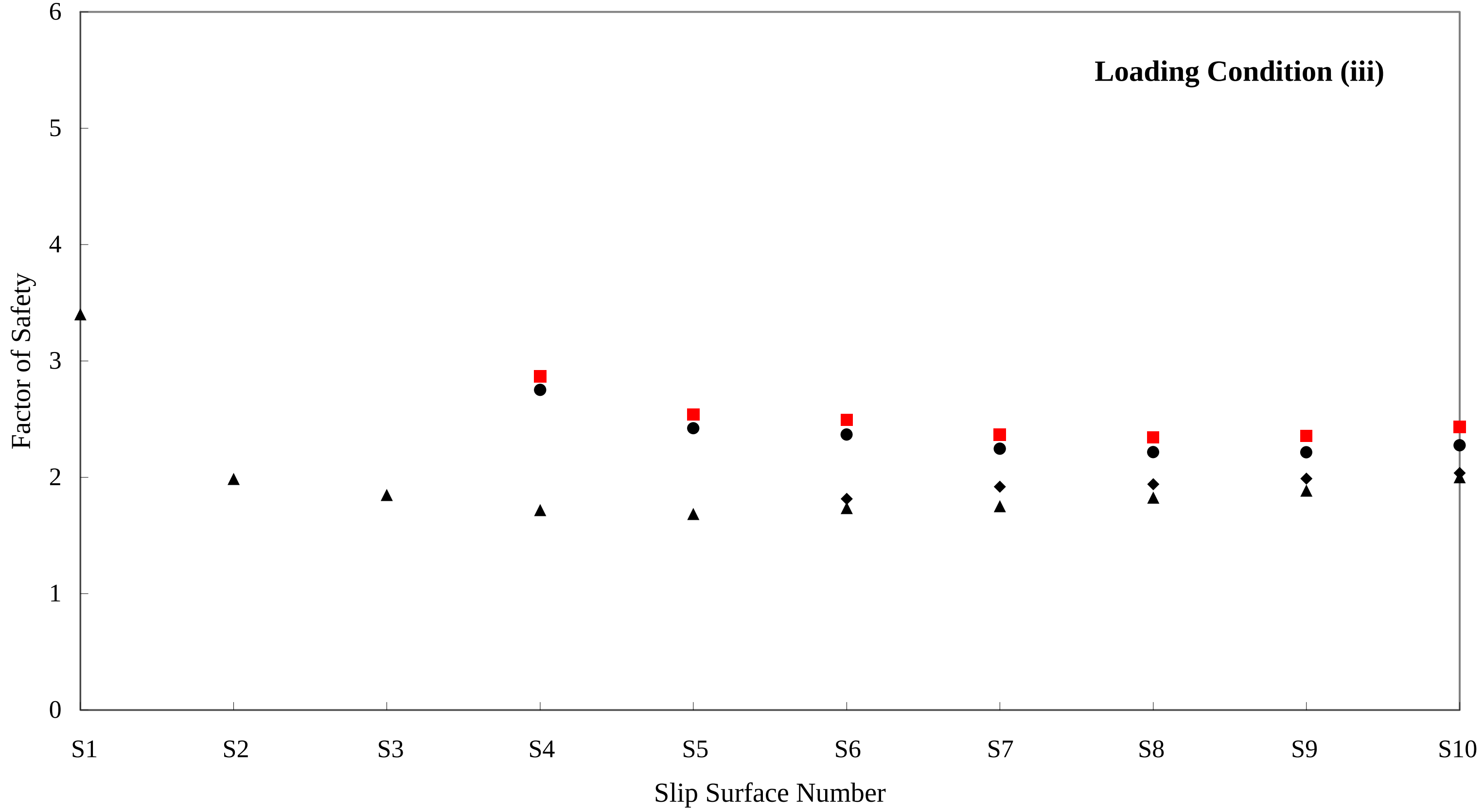

Legend:

◆	Morgenstern-Price	■	Janbu's Simplified with correction factor
▲	Bishop's Simplified	●	Janbu's Simplified with no correction factor

Figure 8 - Factor of Safety versus Slip Surface Number for Loading Condition (iii)

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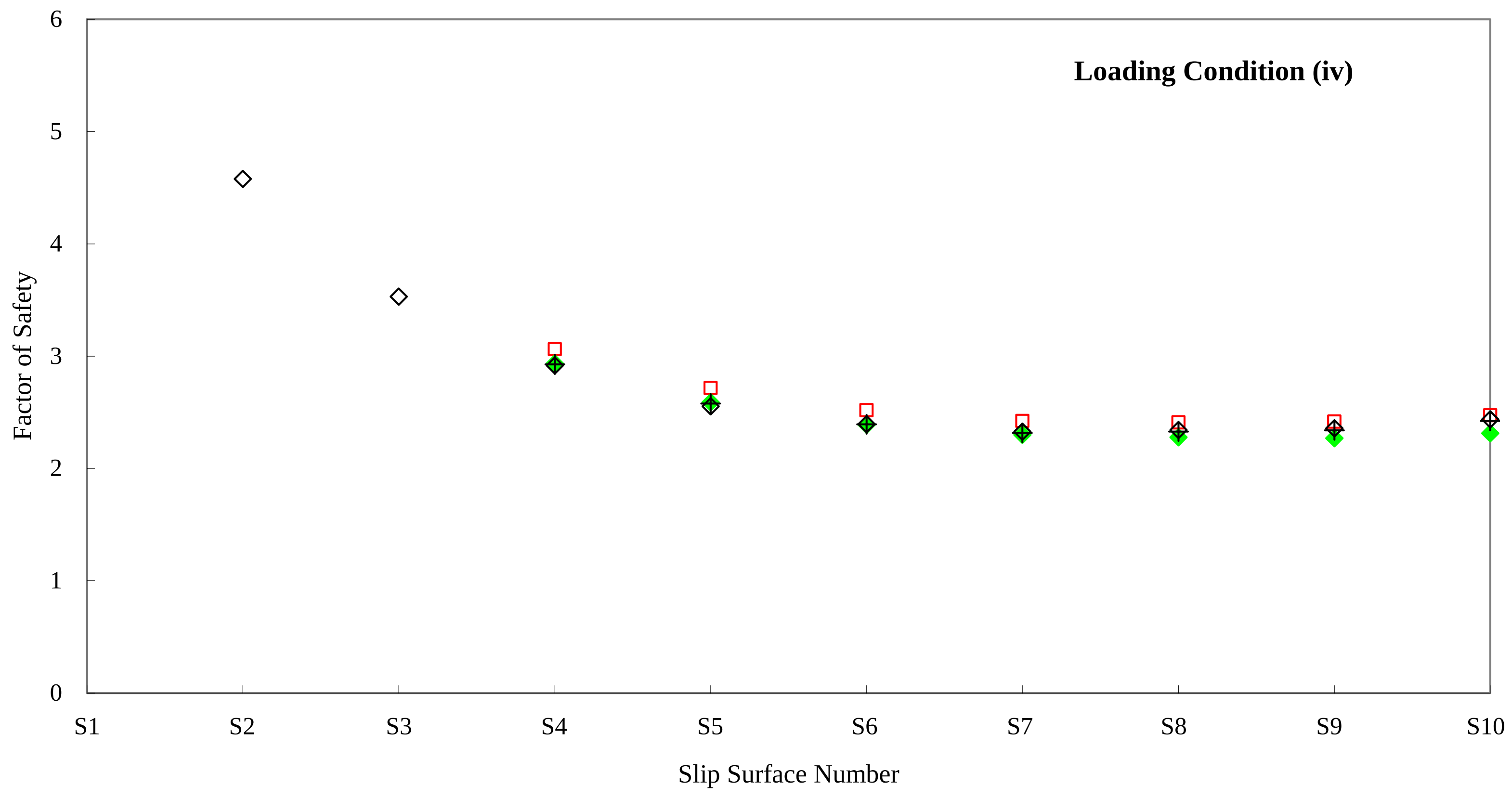

Legend:

+	Morgenstern-Price	□	Janbu's Simplified with correction factor
◇	Bishop's Simplified	◆	Janbu's Simplified with no correction factor

Figure 9 - Factor of Safety versus Slip Surface Number for Loading Condition (iv)

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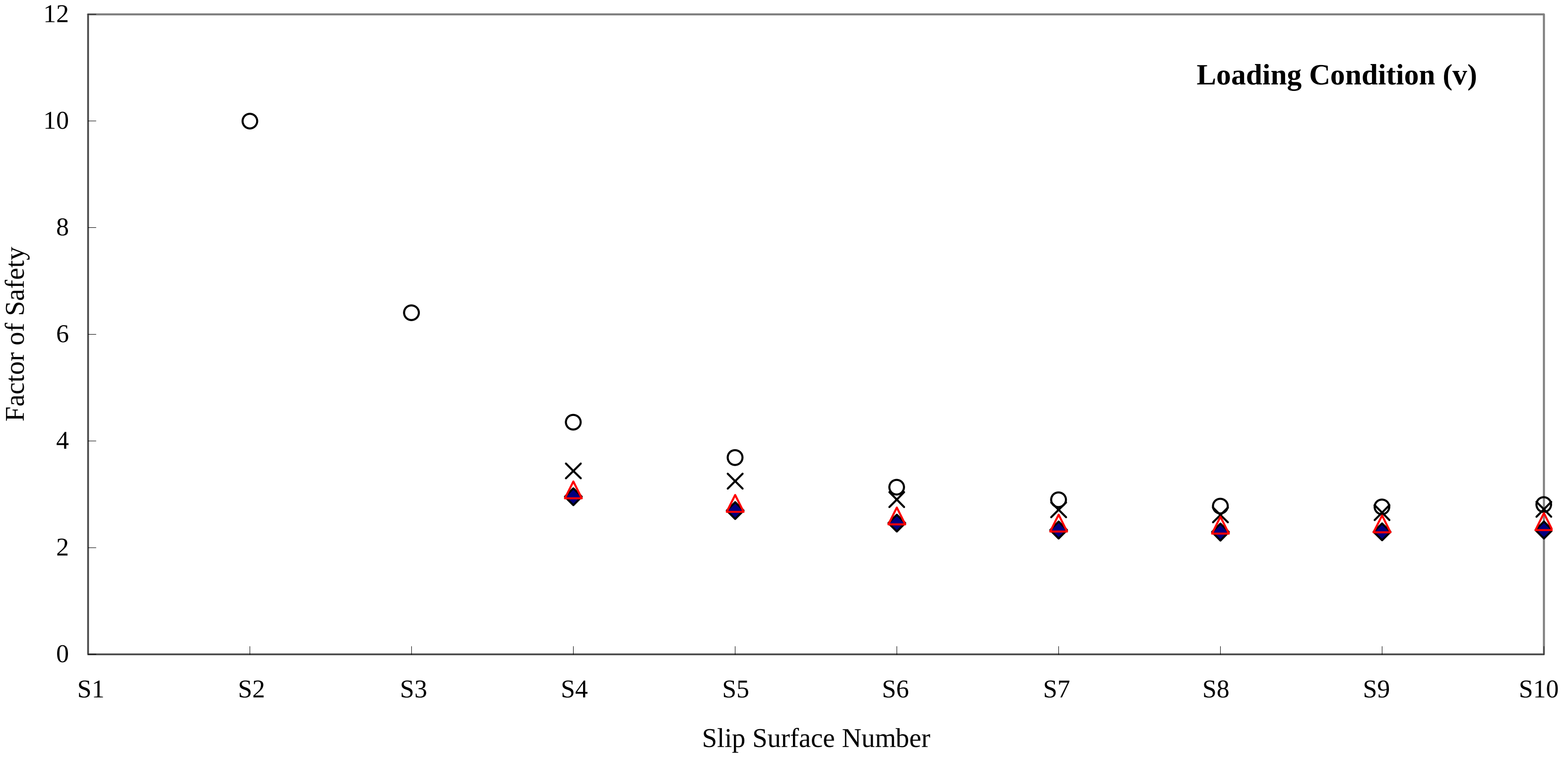

Legend:

×	Morgenstern-Price	△	Janbu's Simplified with correction factor
○	Bishop's Simplified	◆	Janbu's Simplified with no correction factor

Figure 10 - Factor of Safety versus Slip Surface Number for Loading Condition (v)

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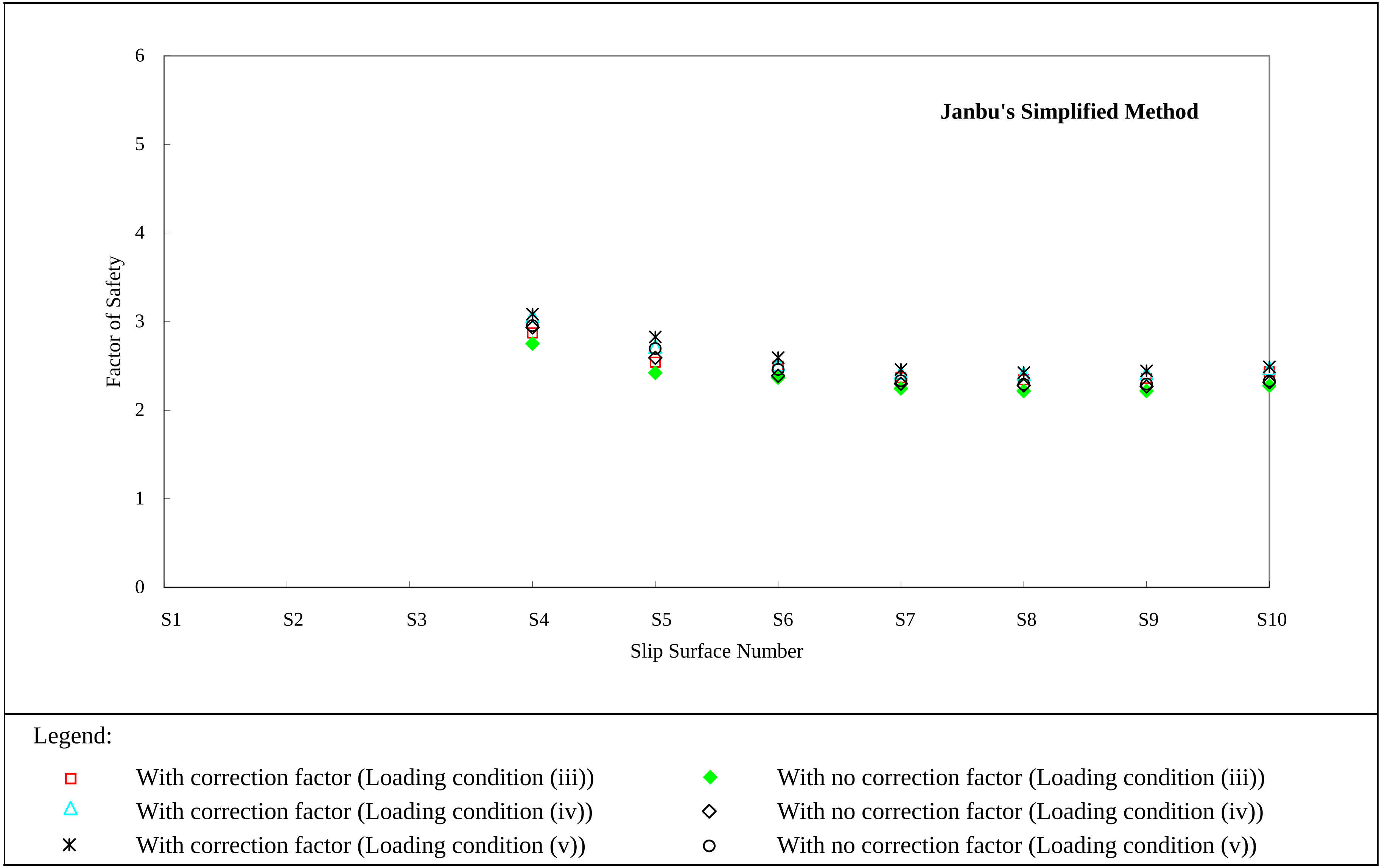

Figure 11 - Factor of Safety Values Computed Using Janbu's Simplified Method for Loading Conditions (iii), (iv) and (v)

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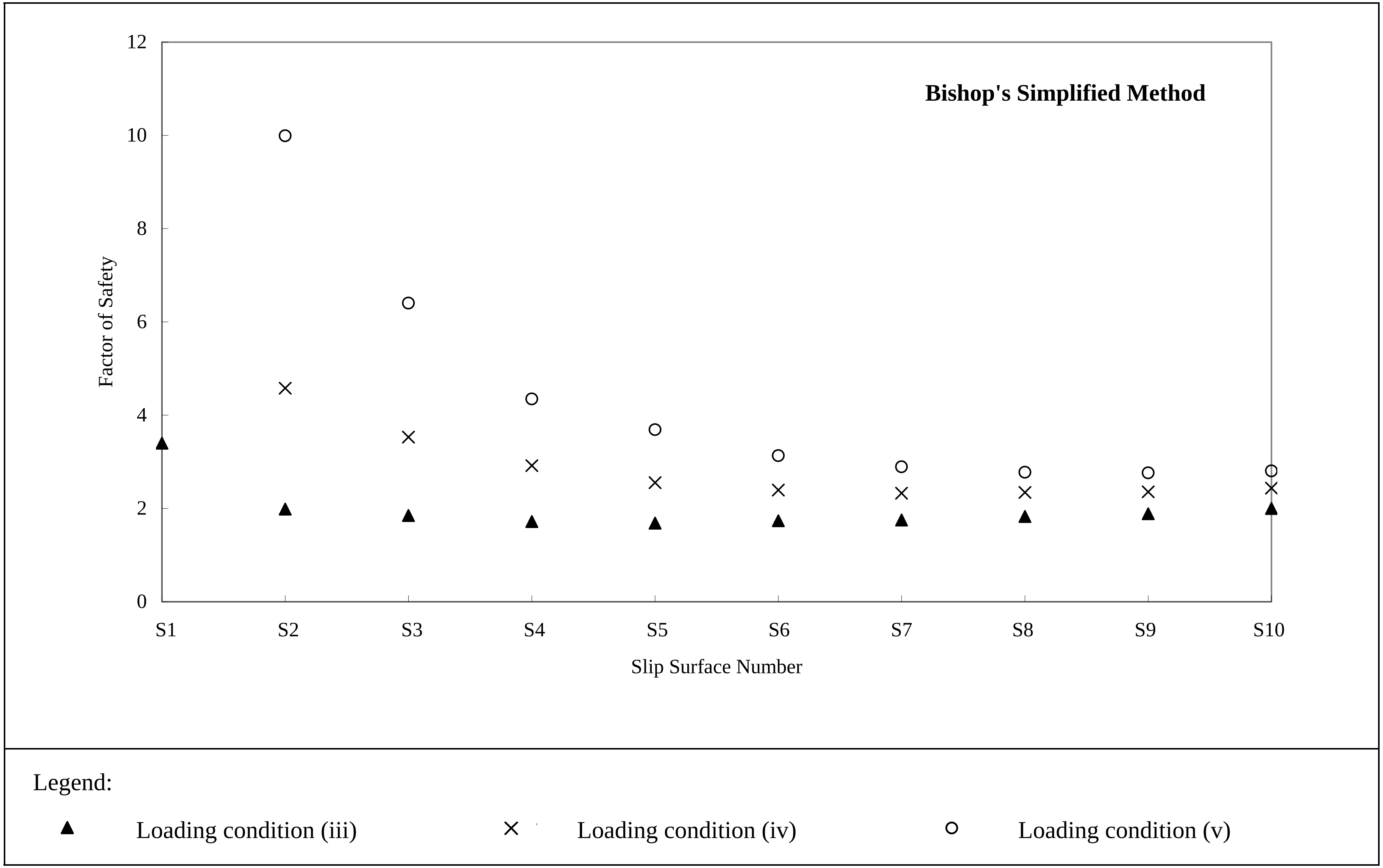

Figure 12 - Factor of Safety Values Computed Using Bishop’s Simplified Method for Loading Conditions (iii), (iv) and (v)

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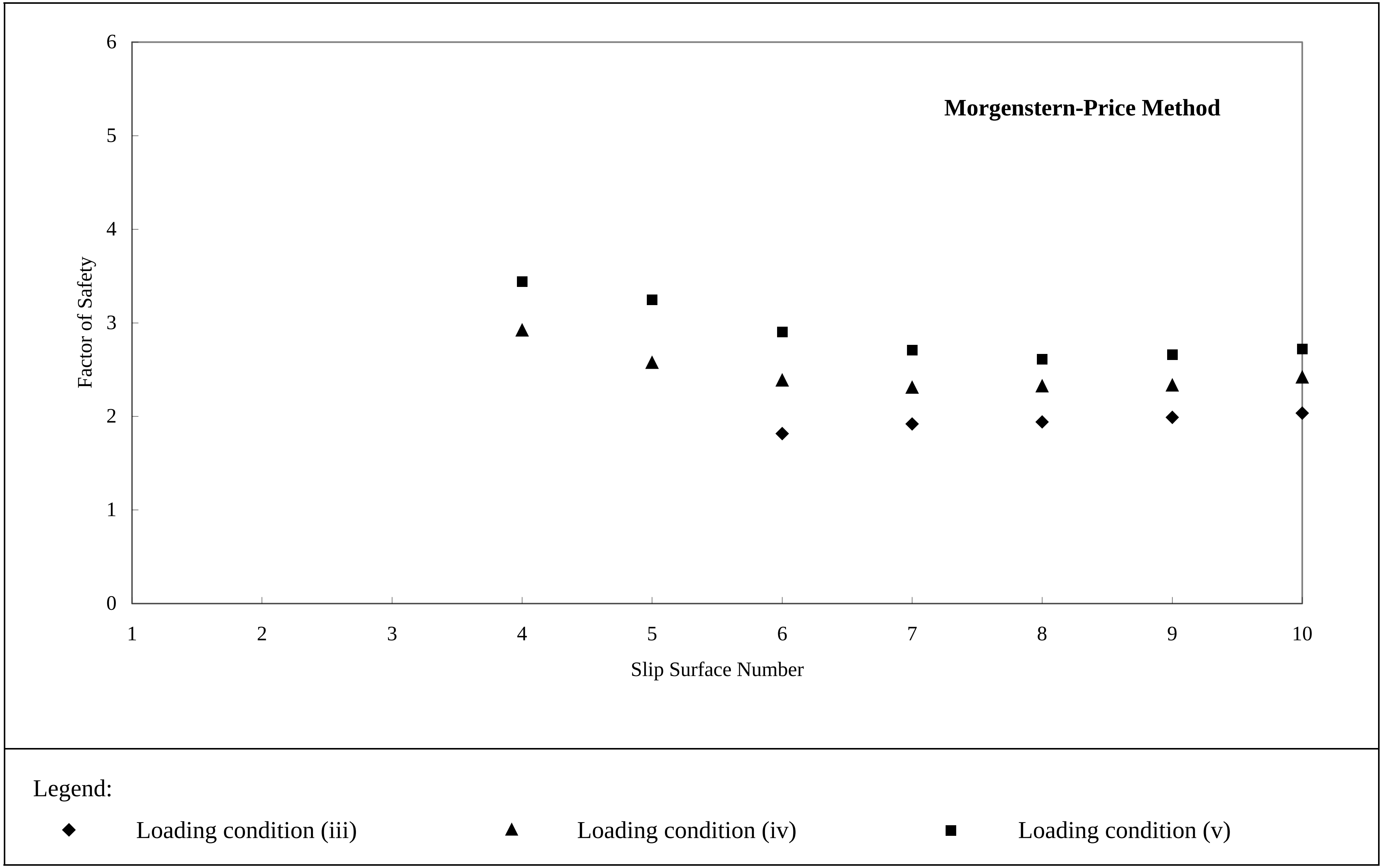

Figure13 - Factor of Safety Values Computed Using Morgenstern-Price Method for Loading Conditions (iii), (iv) and (v)

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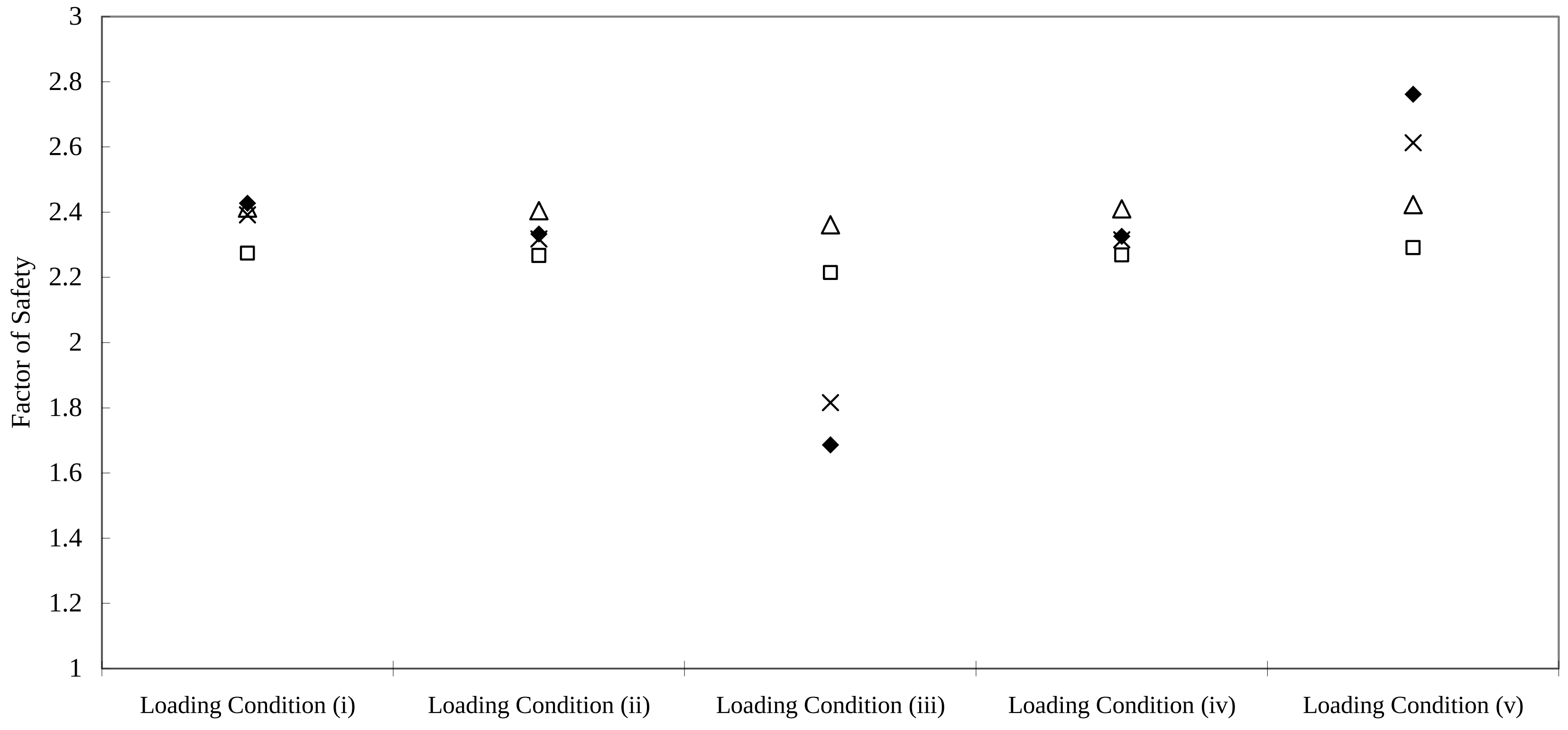

Legend:

◆	Bishop's Simplified	□	Janbu's Simplified with no correction factor
△	Janbu's Simplified with correction factor	×	Morgenstern-Price

Figure 14 - Minimum Factors of Safety Computed Using Different Methods for Loading Conditions (i), (ii), (iii), (iv) and (v)

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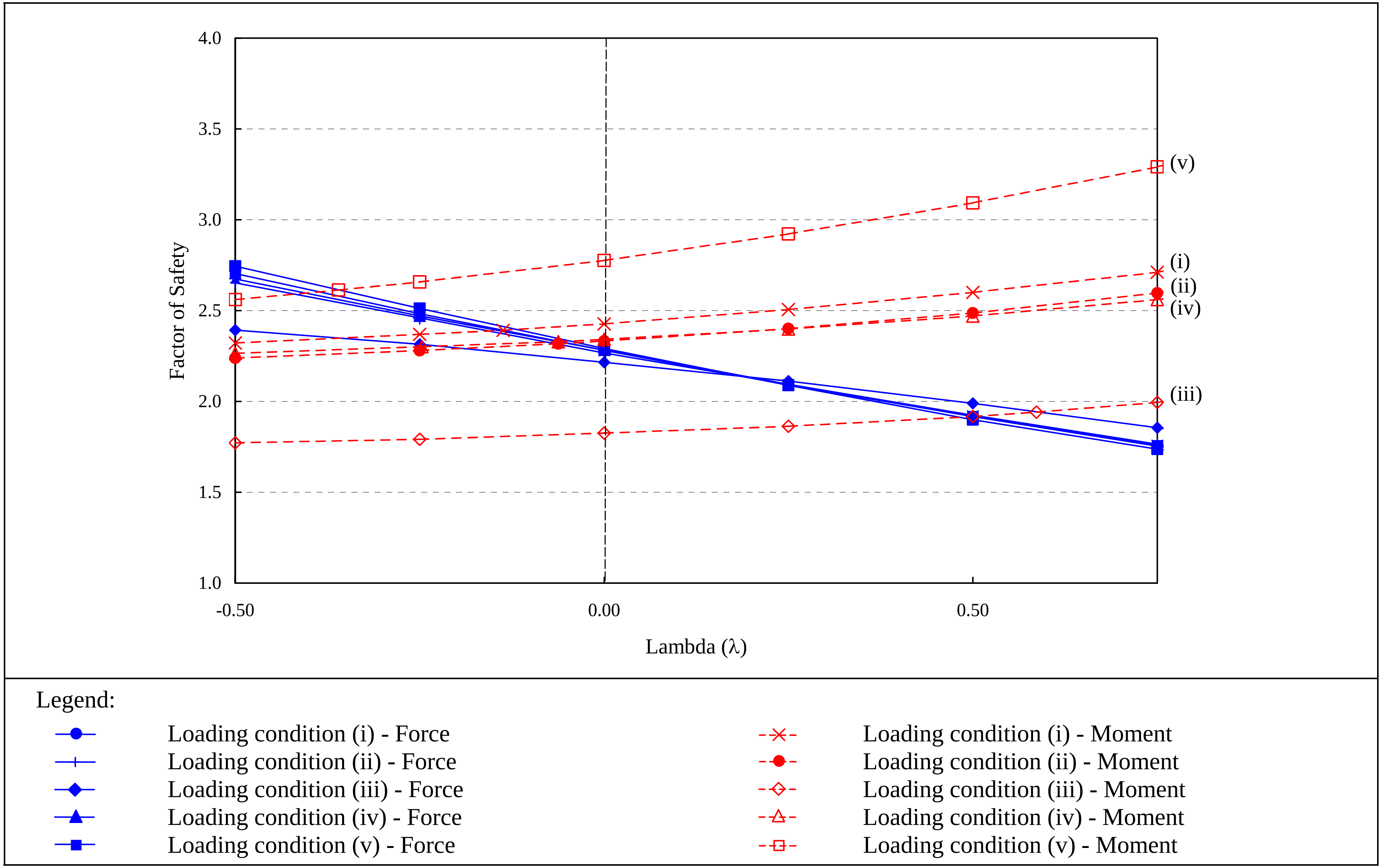

Figure 15 - Factor of Safety (FoS) versus Lambda (λ) Plot for Slip Surface S8 for Loading Conditions (i), (ii), (iii), (iv) & (v) for the Nailed Slope

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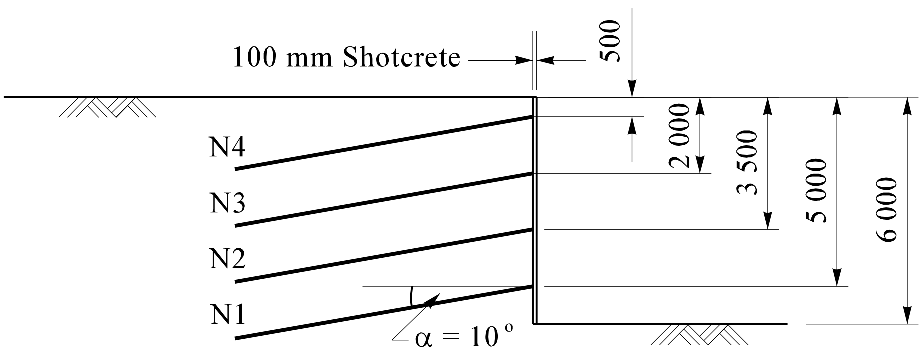

All dimensions are in mm

Soil Nail Properties:

Bar diameter = 40 mm

Soil Nail length = 8 m

Vertical spacing = 1 500 mm

Horizontal spacing = 1 500 mm

Soil Parameters:

$c' = 5$ kN/m^2

$\phi' = 39^\circ$

$\gamma = 19$ kN/m^3

Figure 16 - Geometry and Material Parameters of Nailed Excavation

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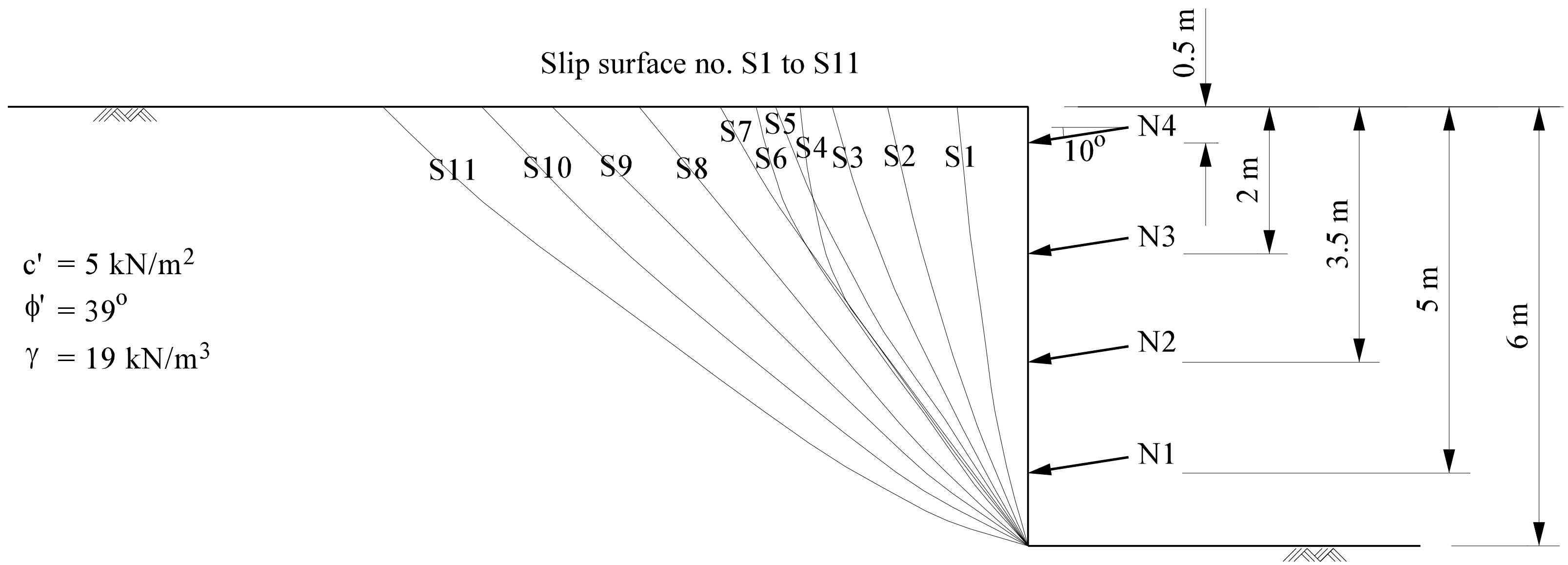

Figure 17 - Locations of Slip Surface for the Nailed Excavation

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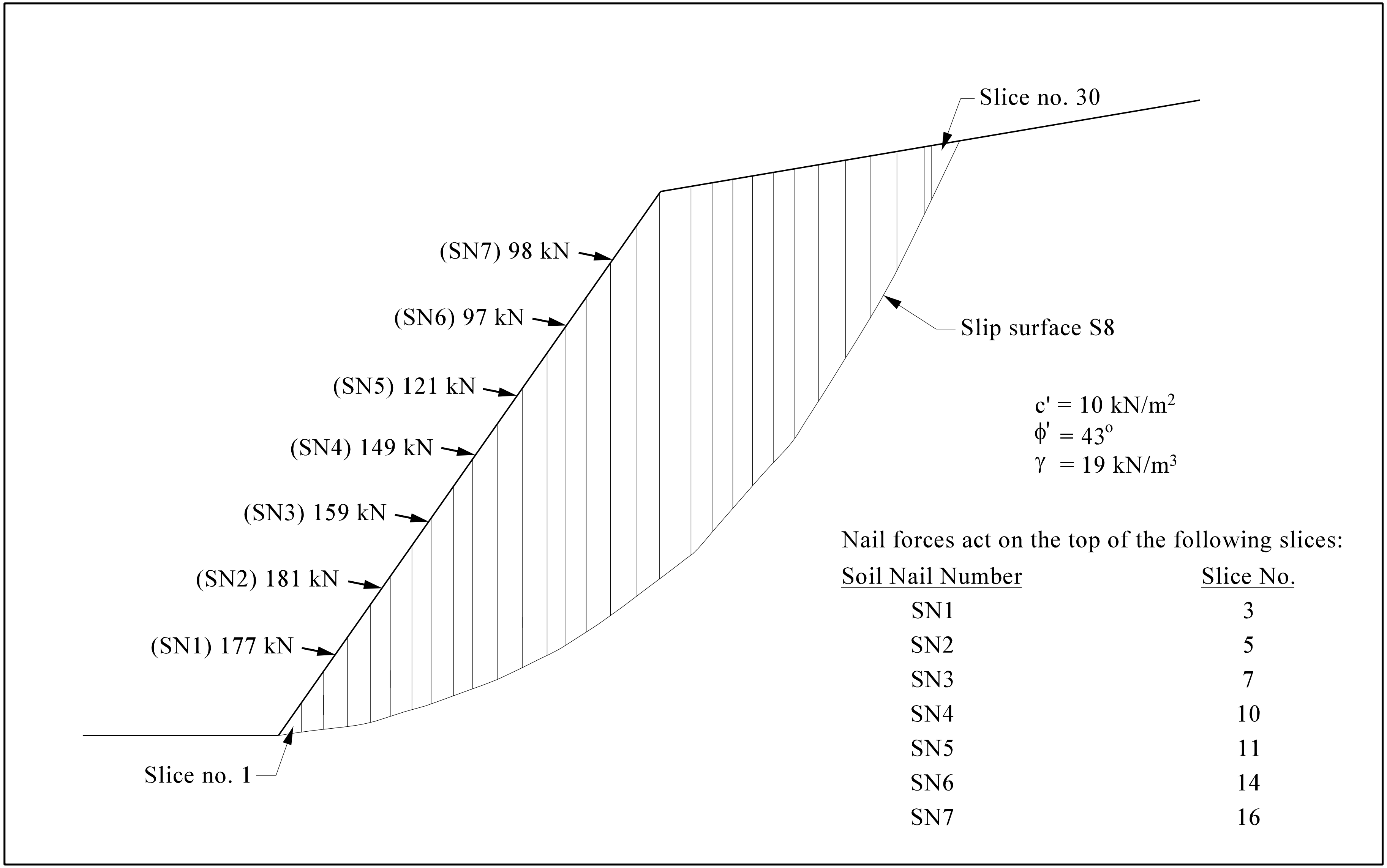

Figure 18 - Different Nail Forces and Slices

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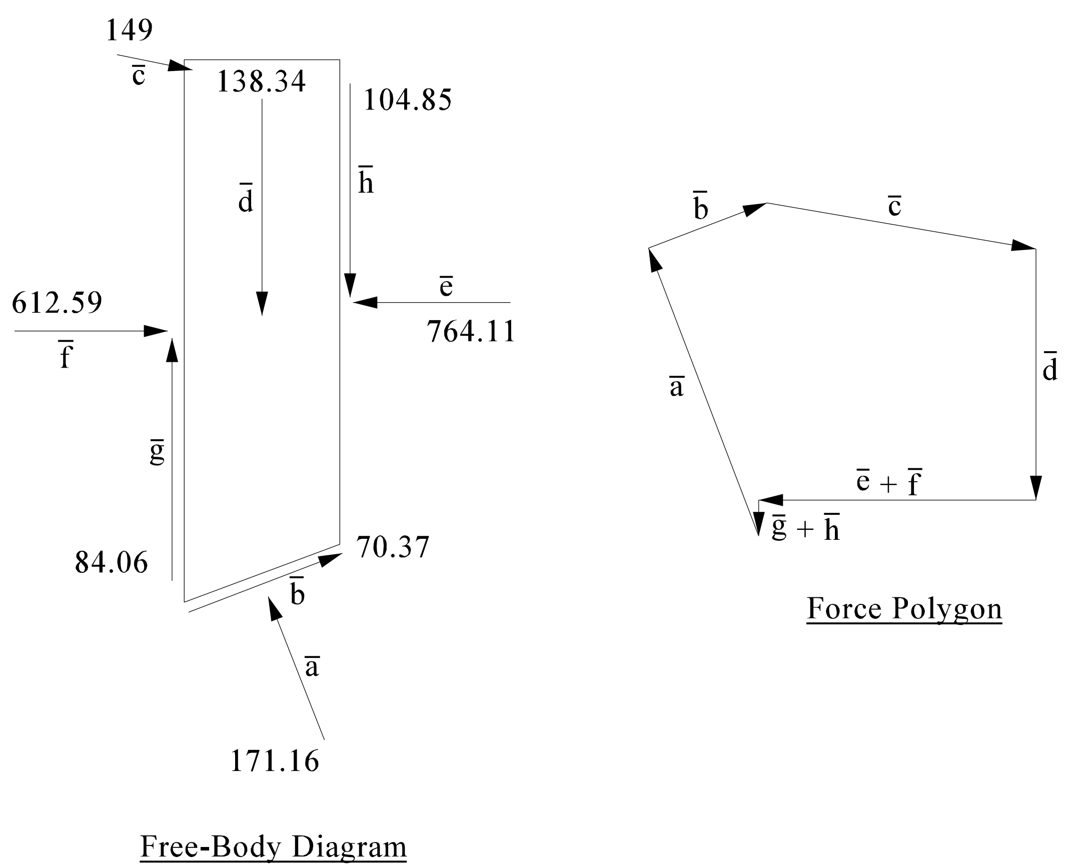
Slice No. 10 - Morgenstern-Price Method



Note: Force unit in kN/m

Figure 19 - Free Body Diagram and Force Polygon for Slice No. 10 When Different Nail Forces Act on the Slope Face

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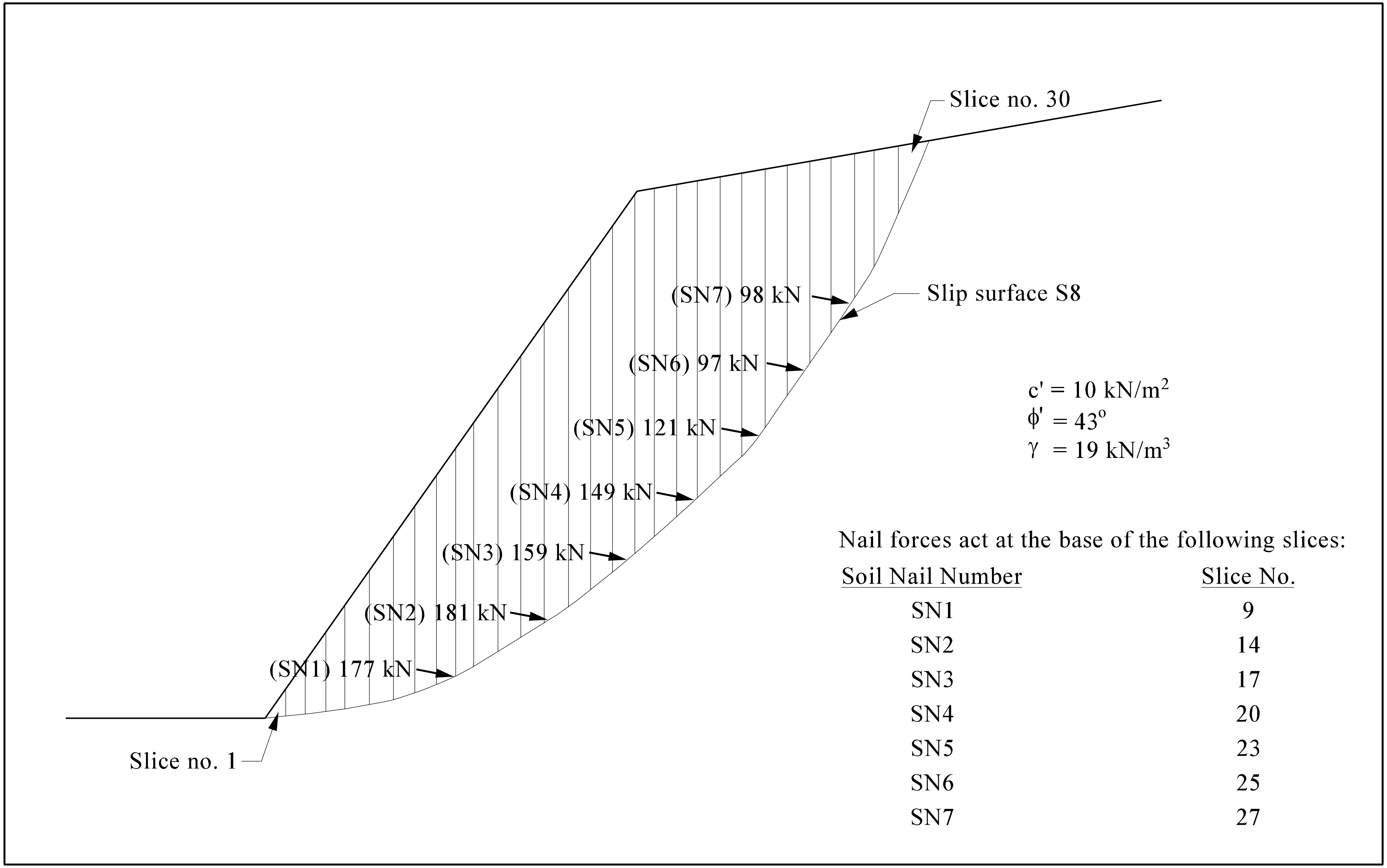

Figure 20 - Different Nail Forces Act on the Slip Surface S8

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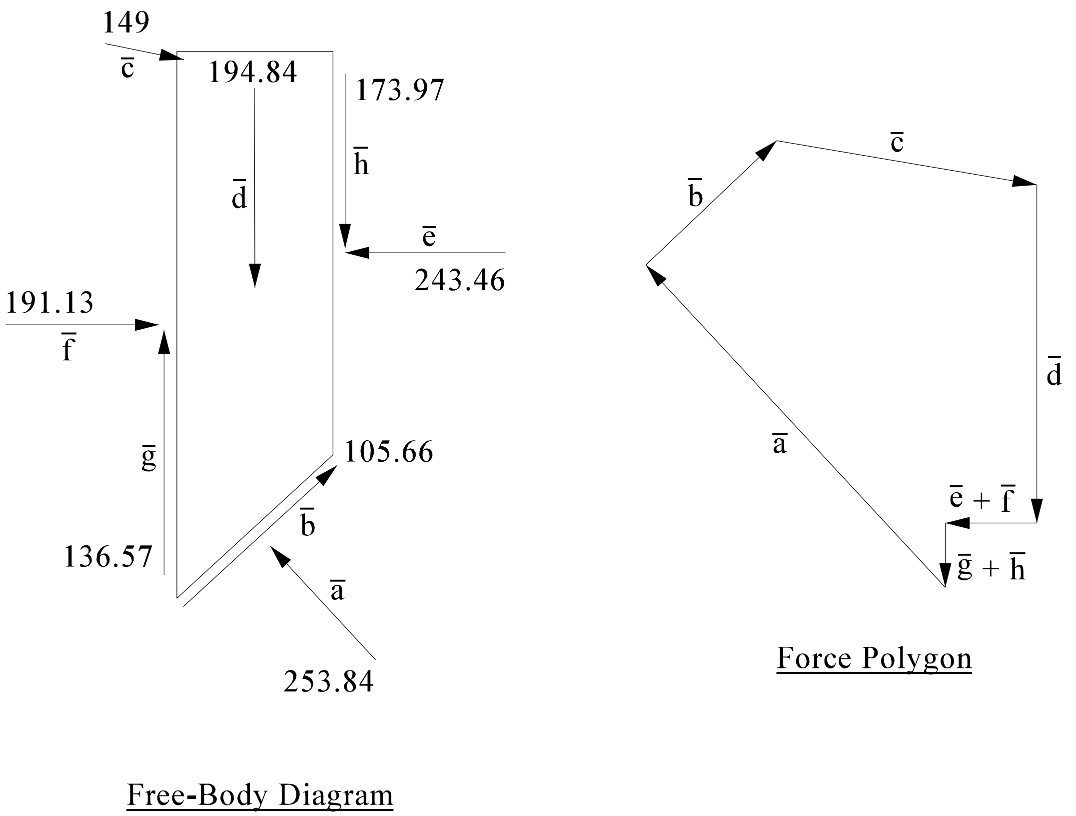
Slice No. 20 - Morgenstern-Price Method



Note: Force unit in kN/m

Figure 21 - Free Body Diagram and Force Polygon for Slice No. 20 When Different Nail Forces Act on the Slip Surface S8

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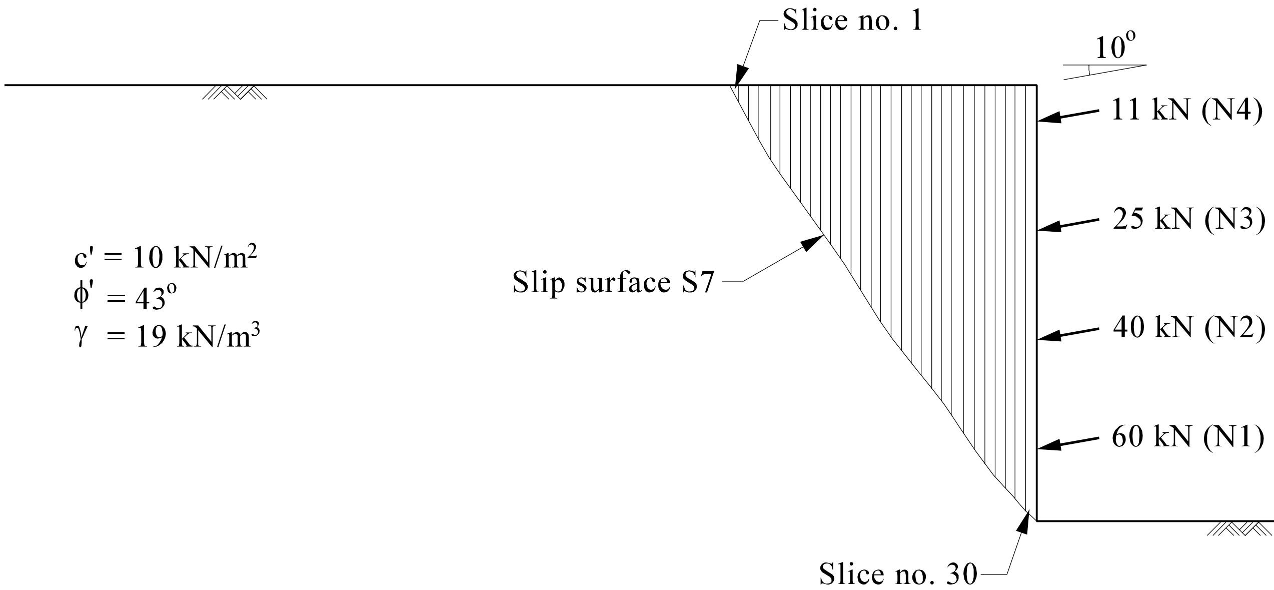

Figure 22 - Different Nail Forces Act on the Excavation Face

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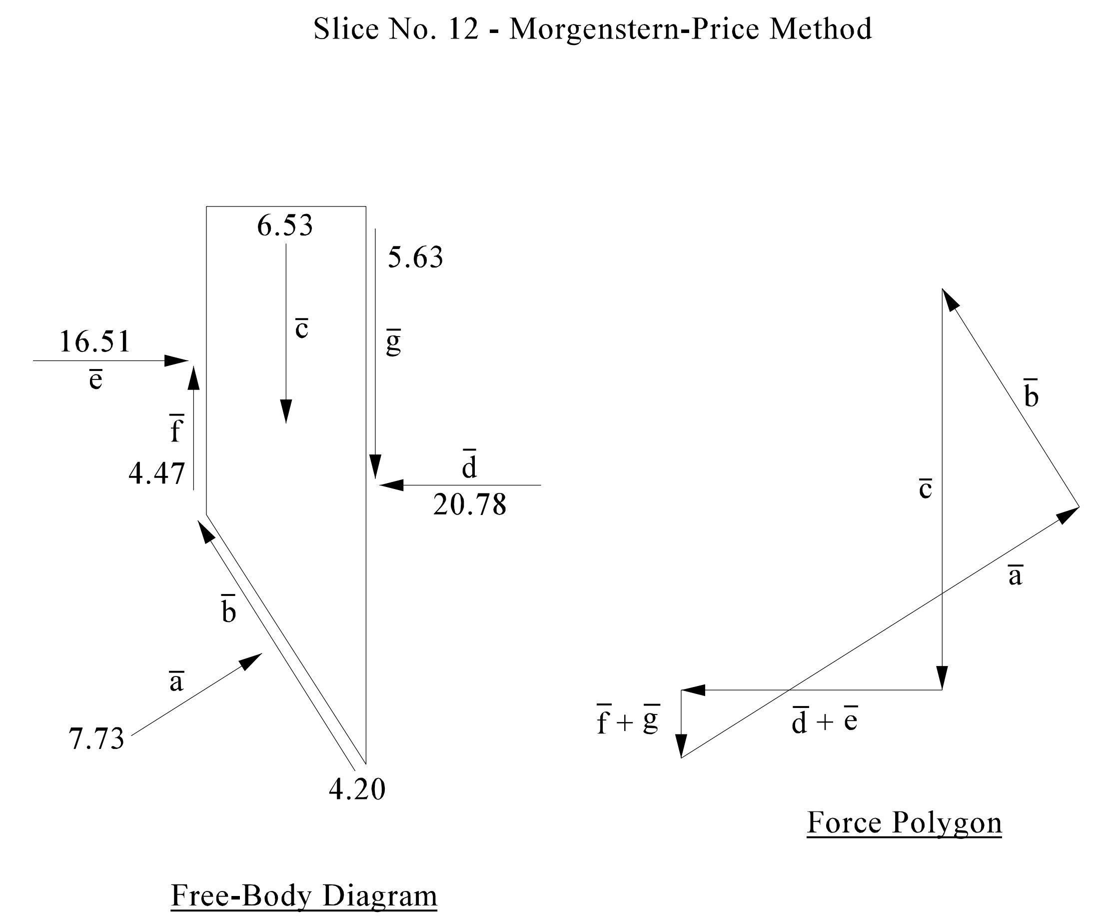

Note: Force unit in kN/m

Figure 23 - Free Body Diagram and Force Polygon for Slice No. 12 When Different Nail Forces Act on the Slip Surface S7

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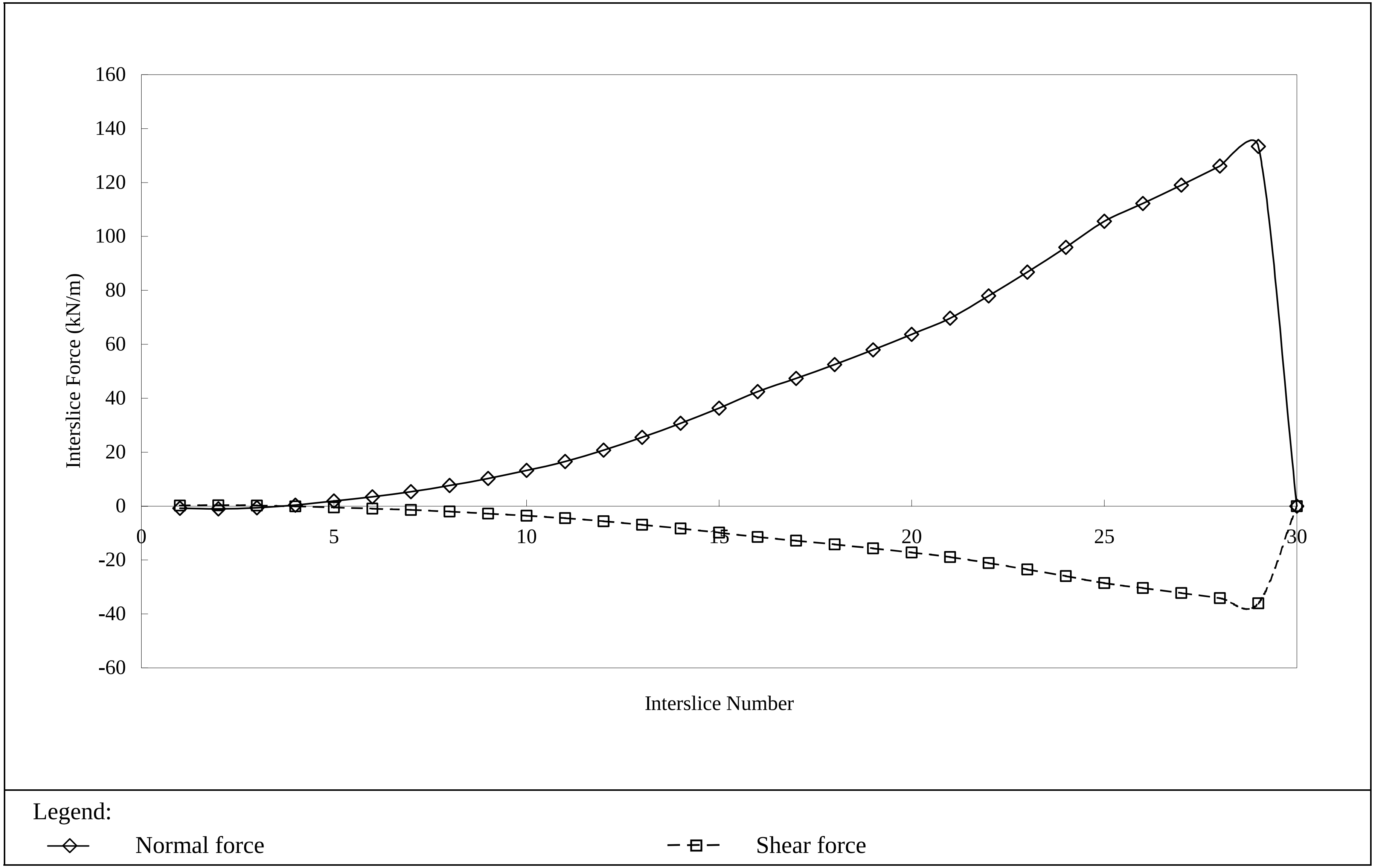

Figure 24 - Distribution of Inter-slice Shear and Normal Forces When Different Nail Forces Act on the Slope Face

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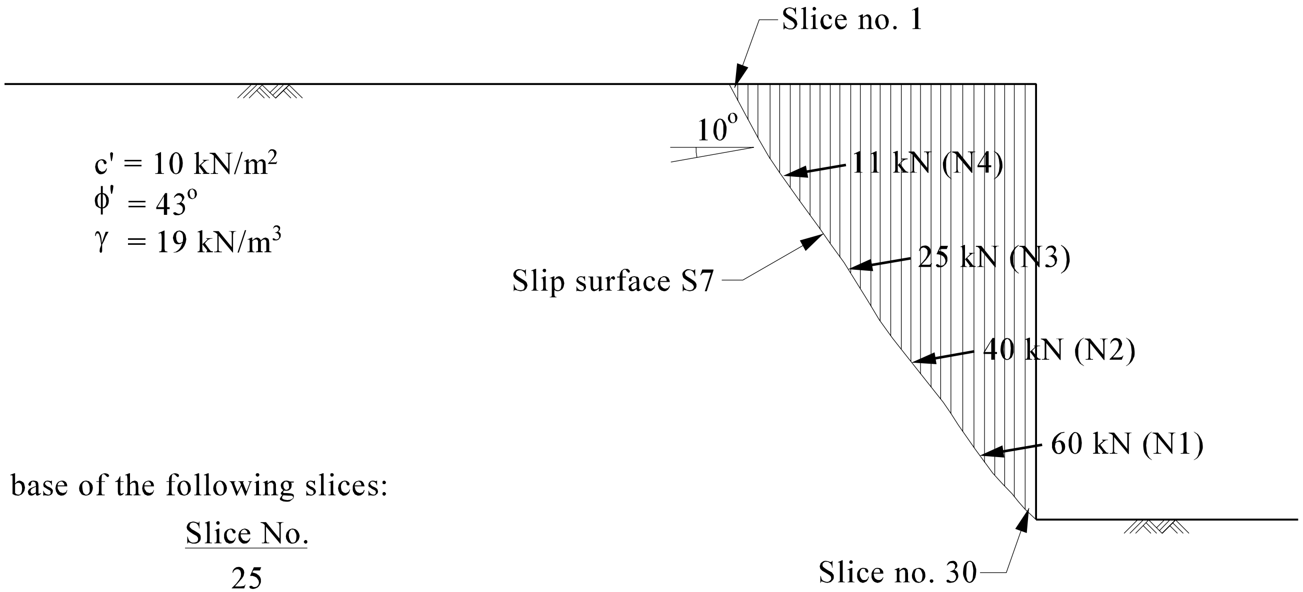

Nail forces act on the base of the following slices:

Soil Nail Number	Slice No.
N1	25
N2	19
N3	12
N4	6

Figure 25 - Different Nail Forces Act on the Slip Surface S7

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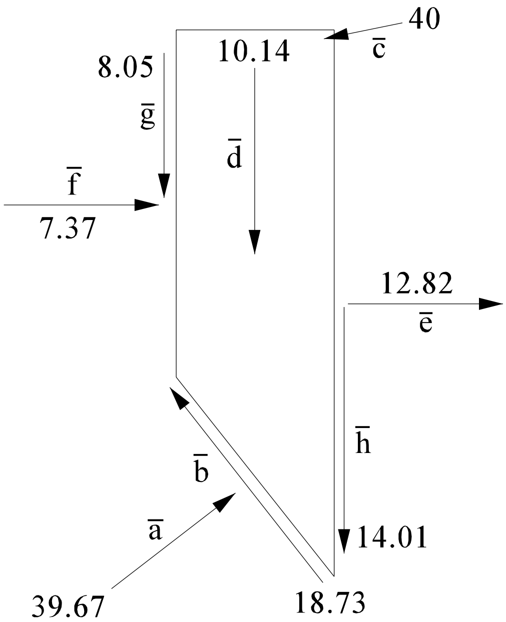
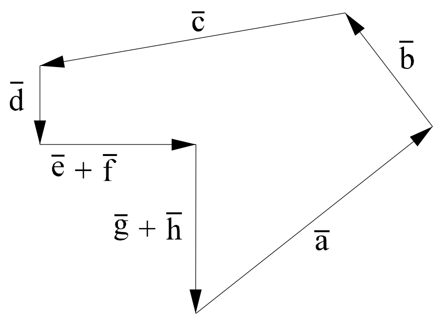
Slice No. 19 - Morgenstern-Price Method



Free-Body Diagram



Force Polygon

Note: Force unit in kN/m

Figure 26 - Free Body Diagram and Force Polygon for Slice No. 19 When Different Nail Forces Act on the Slip Surface S7

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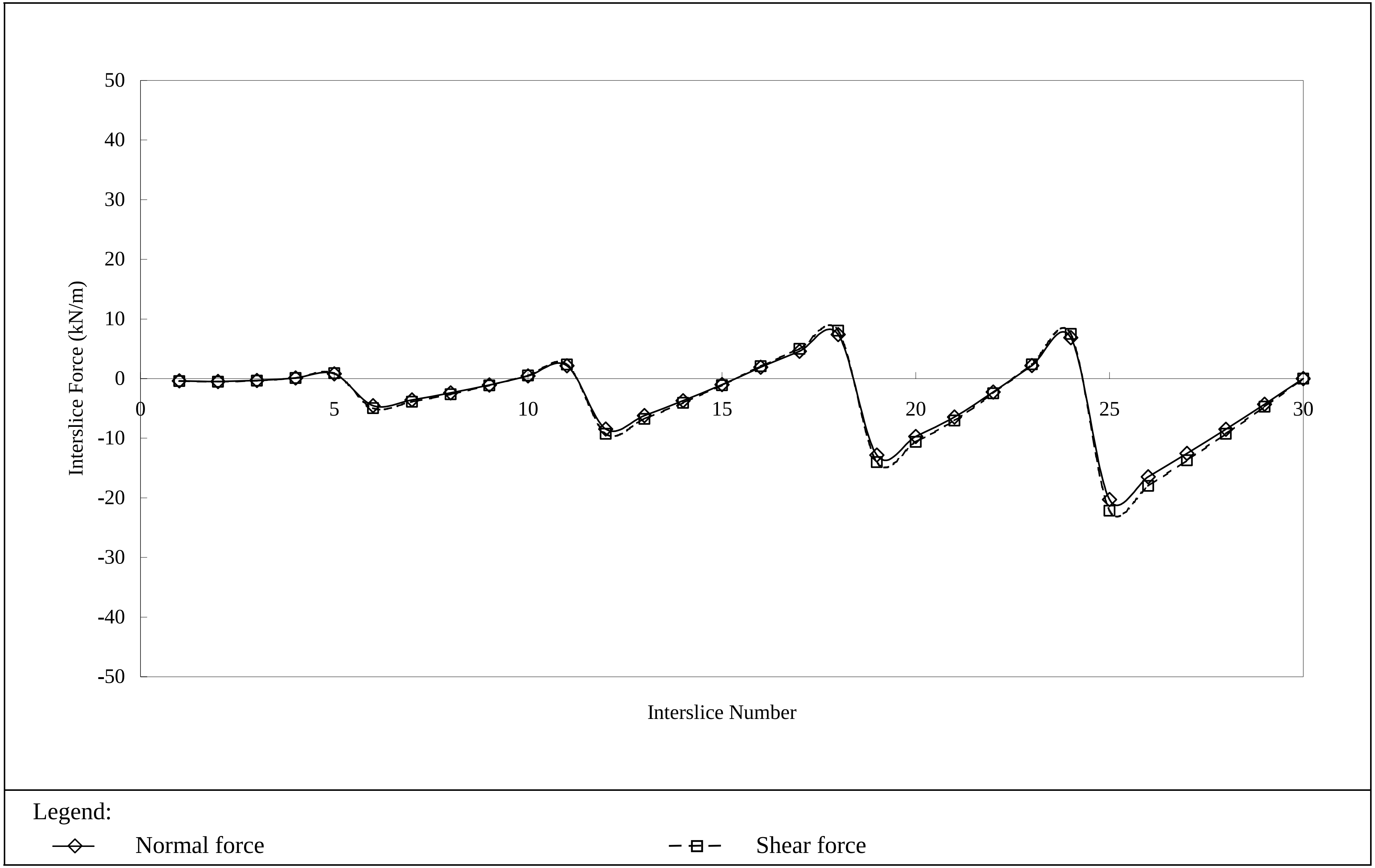

Figure 27 - Distribution of Inter-slice Shear and Normal Forces When Different Nail Forces Act on the Slip Surface S7

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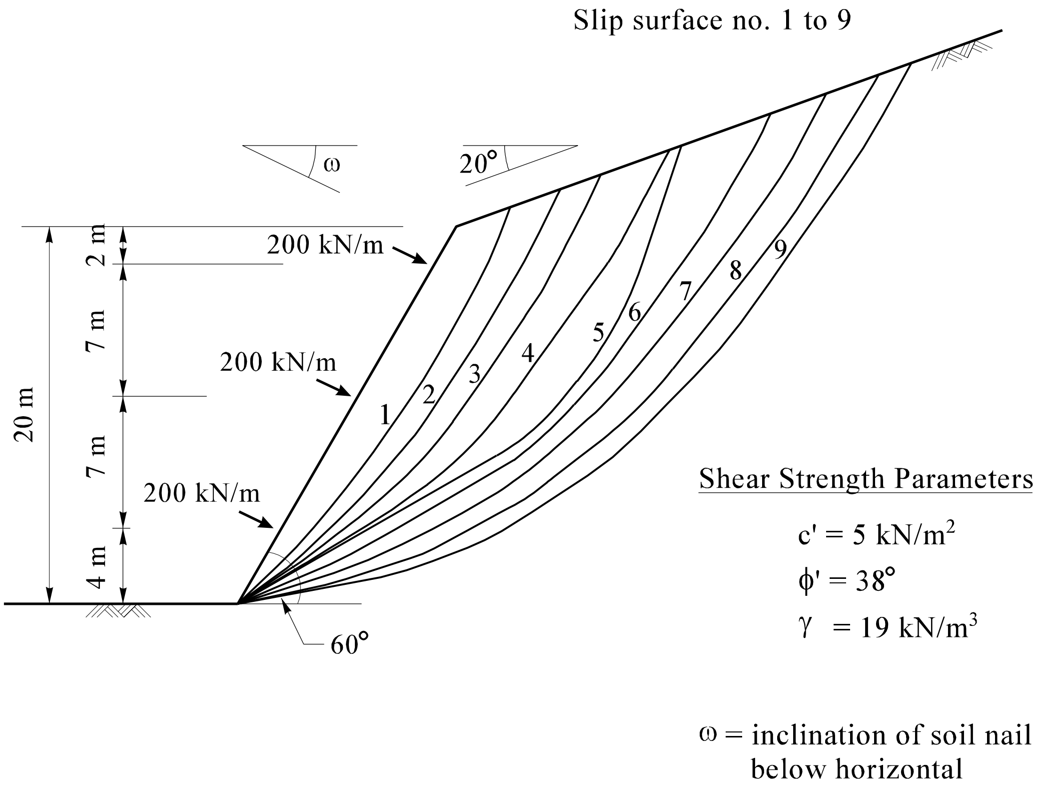

Figure 28 - Nailed Slope Model for the Comparison of Design Using Horizontal Component of Nail Force to that Using Nail Force along the Same Inclination of Soil Nail

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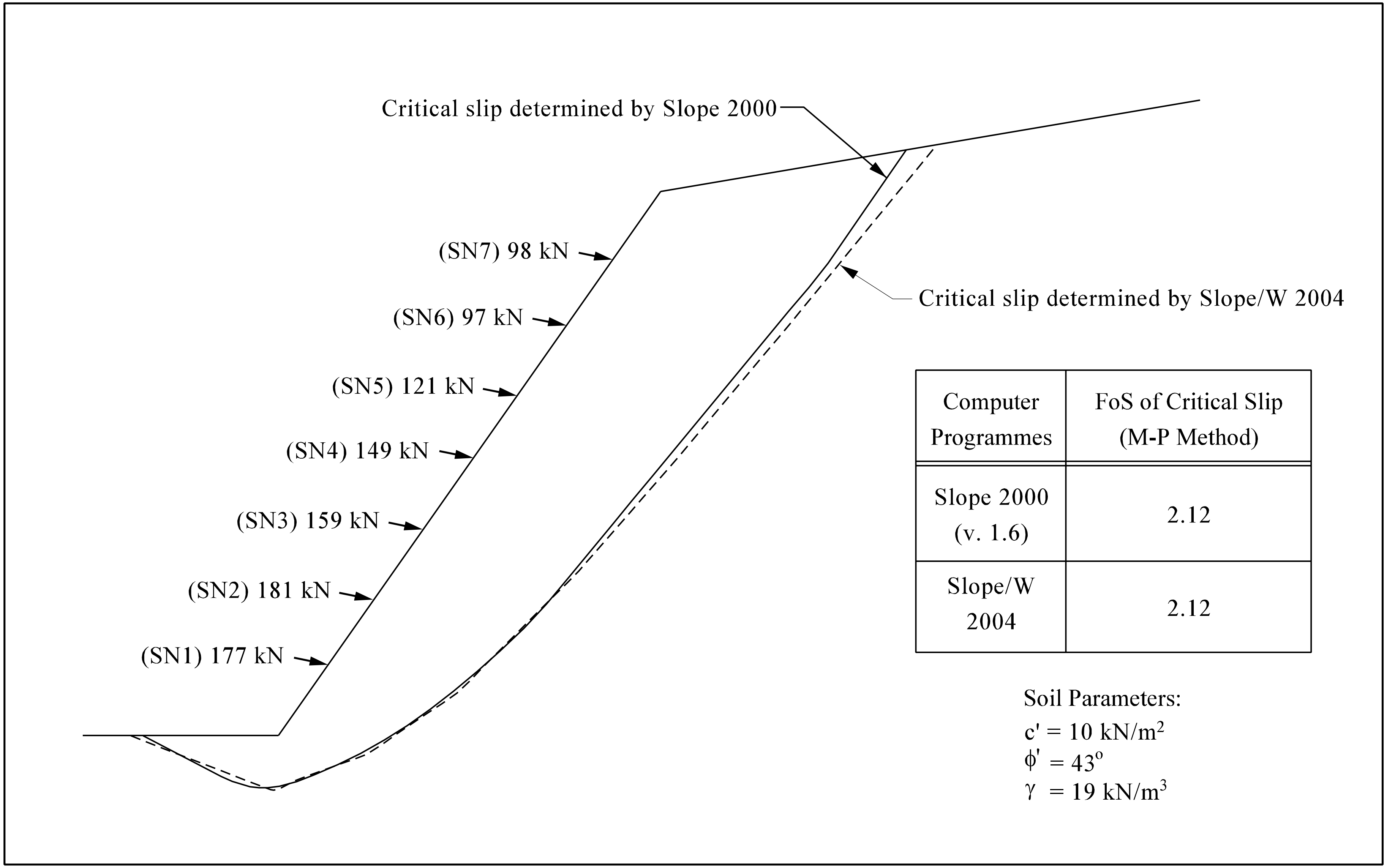

Computer Programmes	FoS of Critical Slip (M-P Method)
Slope 2000 (v. 1.6)	2.12
Slope/W 2004	2.12

Soil Parameters:

$c' = 10$ kN/m^2

$\phi' = 43^o$

$\gamma = 19$ kN/m^3

Figure 29 - Optimization Using Different Computer Programmes

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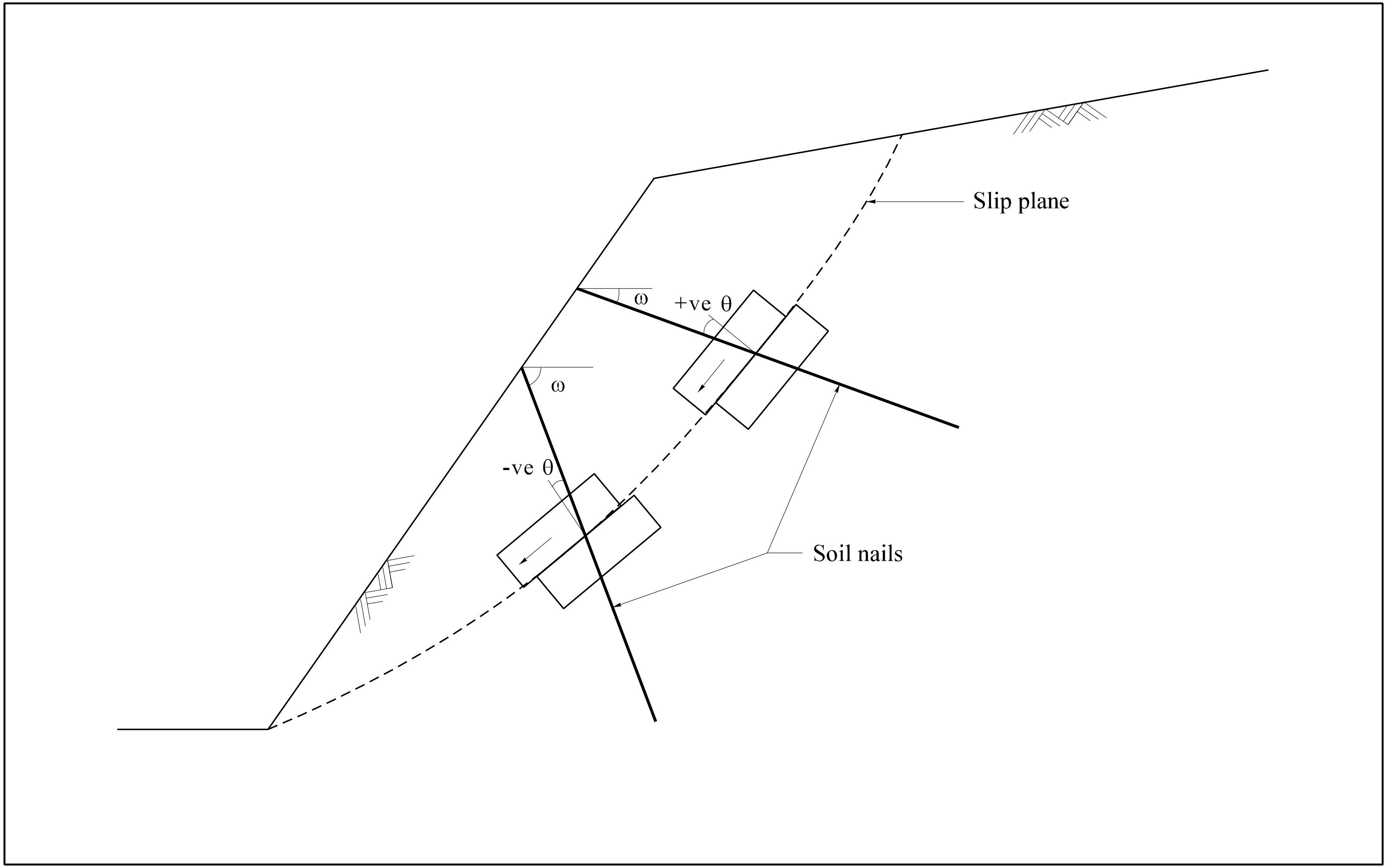

Figure 30 - Relationship between Nail Inclination ω and Nail Orientation θ

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APPENDIX A

METHOD OF SLICES

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CONTENTS

A.1 METHODS OF SLICES COMMONLY USED IN SOIL NAIL DESIGN

Four methods of slices are reviewed and compared in this study. They are the Janbu's Simplified (JS) method, the Janbu's Generalized (JG) method, the Bishop's Simplified (BS) method and the Morgenstern-Price (M-P) method in respect of soil nail design.

Figure A1 shows a typical slice in a potential sliding mass and the forces acting on the slice. Both the BS and JS methods assume that the inter-slice forces are horizontal. The BS method satisfies vertical force equilibrium for each slice and overall moment equilibrium about the centre of circular slip surface. The JS method satisfies vertical force equilibrium for each slice, as well as overall horizontal force equilibrium for the entire slide mass. The JS method often produces factors of safety that are smaller than those obtained by more rigorous methods that satisfy both force and moment equilibrium. To account for this, Janbu et al (1973) proposed the empirical correction factors for compensating the effect due to the assumption of neglecting inter-slice shear forces as shown in Figure A2. The correction factors generally increase the factor of safety up to approximately 10%.

The JG method includes the effect of inter-slice forces by making an assumption regarding the point at which the resultant inter-slice force acts on each slice. The resultant inter-slice force is often assumed to act at the one-third of the inter-slice height above the slip surface. A line passing through the resultants by the slice vertical sides is the line of thrust. The stability calculations are sensitive to the location of the line of thrust and the method does not always produce a stable numerical solution (Li, 1986; Abramson et al, 2002). The JG method satisfies both the vertical and horizontal force equilibrium but it satisfies moment equilibrium in only an approximate way. As such, there has been some debate as to whether this method satisfies complete equilibrium or only force equilibrium.

The M-P method (Morgenstern & Price, 1965) assumes that the inter-slice shear forces (X) are related to the inter-slice normal forces (E) as:

$$X = E\lambda \ f(x) \quad \text{..(A1)}$$

where f(x) is a function, λ is the percentage of the function used. Figure A3 shows some typical functions. This method satisfies the vertical and horizontal force equilibrium, as well as the moment equilibrium.

A general limit equilibrium (GLE) formulation was developed by Fredlund in the 1970s (Fredlund & Krahn 1977; Fredlund et al 1981). The formulation is useful for understanding the reasons for the differences among the JS method, the BS method and the M-P method. As such, its fundamental principle is described here.

The GLE formulation uses Equation (A1) to relate the inter-slice normal force and the inter-slice shear force. The GLE formulation is based on two factors of safety equations. One equation gives the factor of safety for moment equilibrium (F_m), and the other one gives the factor of safety for horizontal force equilibrium (F_f).

Referring to Figure A1, the moment equilibrium equation for F_m and the horizontal force equilibrium equation for F_f are:

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$$F_m = \frac{\Sigma[c'\beta R + (N - u\beta)R \tan\phi']}{\Sigma Wx - \Sigma Nf \pm \Sigma Dd} \quad \text{.....................................(A2)}$$

and:

$$F_f = \frac{\Sigma[c'\beta\cos\alpha + (N - u\beta)\tan\phi'\cos\alpha]}{\Sigma N\sin\alpha - \Sigma D\cos\omega} \quad \text{................................(A3)}$$

where N is the normal force acting at the base of a slice surface. It is given by:

$$N = \frac{W + (X_R - X_L) - \dfrac{c'\beta\sin\alpha - u\beta\sin\alpha\tan\phi'}{F} + D\sin\omega}{\cos\alpha + \dfrac{\sin\alpha\tan\phi'}{F}} \quad \text{...............(A4)}$$

The denominator of Equation (4) is often referred to as m_α:

$$m_\alpha = \cos\alpha + \frac{\sin\alpha\ \tan\phi'}{F} \quad \text{...(A5)}$$

The definitions of the variables in Equations (A2) to (A5) are given in Figure A1. The derivations of the equations can be found in Fredlund et al (1981) or Krahn (2003).

The GLE formulation calculates F_m and F_f for a range of λ values. With these values, a graph of factor of safety versus λ, such as that shown in Figure A4, can be plotted. As indicated above, both the BS method and JS method ignore inter-slice shear forces. Having no inter-slice shear forces means that λ is zero. Since the BS method satisfies only moment equilibrium, the Bishop factor of safety falls on the moment curve in Figure A4 where λ is zero. Similarly, as the JS method only satisfies overall horizontal force equilibrium, the Janbu factor of safety (without correction factor) falls on the force curve in Figure A4 where λ is zero. The M-P factor of safety is determined at the point where the F_m and F_f curves intersect (see Figure A4). The factor of safety satisfies both moment and force equilibrium at this point. If a constant inter-slice function f(x) is used, the intersection point is the same as that computed using Spencer’s method (Spencer, 1967).

The factor of safety versus λ plots are mirrored for right-left problems, as shown in Figure A5. The safety factors are the same, but λ has the opposite sign. This is required to keep all the forces in the correct direction. The GLE formulation and λ plots enable engineers to understand what happens behind the scenes.

For methods that satisfy moment equilibrium, a point needs to be established about which to take moments. This is called the Point of Axis of Moments (Axis Point) and this has to be specified in the calculation (Krahn, 2004).

According to Fredlund et al (1992), the factor of safety calculations using methods that satisfy complete static equilibrium are not affected by the position of the Axis Point. However, for simplified methods that satisfy moment equilibrium only, such as the BS method, the factor of safety calculations may be sensitive to it if the slip surface is not circular.

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Krahn (2004) suggested that the Axis Point should be in general in a location close to the approximate centre of rotation of the sliding mass. It is usually somewhere above the slope crest and between the extents of the potential sliding mass. This suggestion has been adopted in this study when choosing the location of the Axis Point.

A.2 CHARACTERISTICS OF LIMIT EQUILIBRIUM METHOD OF SLICES

According to Duncan & Wright (1980), all equilibrium methods of slope stability analysis have the following four characteristics in common:

(1) They all use the same definition of the factor of safety (FoS):
FoS = shear strength of soil/shear stress required for equilibrium

The factor of safety is the same at all points along the potential slip surface. This is reasonable only at failure, i.e. when the factor of safety equals to 1.

(2) They all assume that the shear strength parameters of soils are not dependent on the stress-strain characteristics of the soils.

(3) They all use equations of equilibrium to calculate the average value of shear stress along the slip surface, and to calculate the normal stress at the base of the slip surface.

(4) They all use assumptions to supplement the equations of equilibrium in order to handle indeterminacy of the problem.

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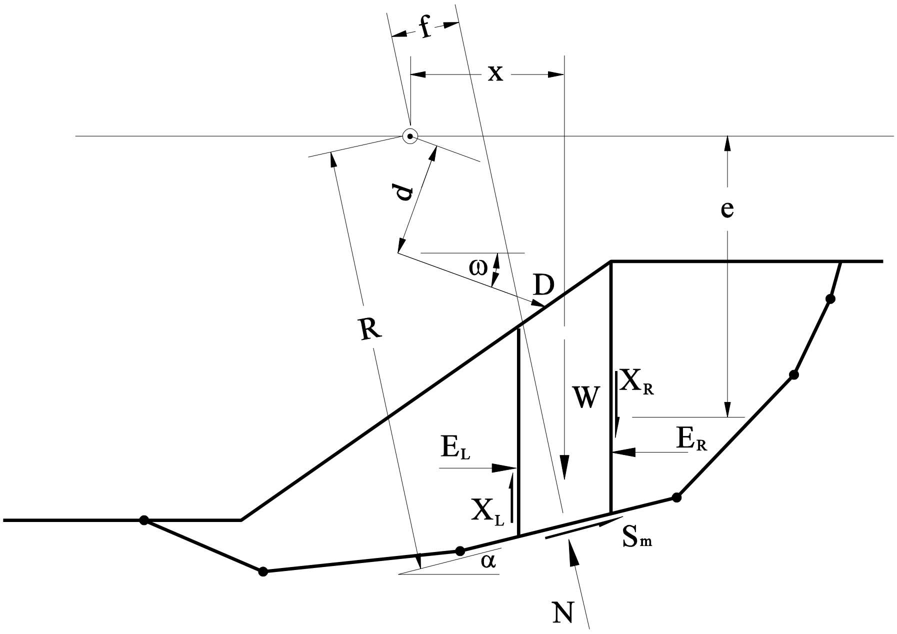
LIST OF FIGURES



Soil Strength Parameters:

s	=	Shear Strength
c'	=	Effective Cohesion
ϕ'	=	Effective Angle of Internal Friction
u	=	Pore-water Pressure

Slice Parameters:

W	=	The total weight of a slice of width b and height h
N	=	The total normal force on the base of the slice
S_m	=	The shear force mobilized on the base of each slice
E	=	The horizontal interslice normal forces. Subscripts L and R designate the left and right sides of the slice, respectively
X	=	The vertical interslice shear forces. Subscripts L and R define the left and right sides of the slice, respectively
D	=	An external line load
R	=	The radius for a circular slip surface or the moment arm associated with the mobilized shear force, s_m for any shape of slip surface
σ_n	=	$\frac{N}{\beta}$ = average normal stress at the base of each slice
β	=	The base length of each slice
f	=	The perpendicular offset of the normal force from the center of rotation or from the center of moments
x	=	The horizontal distance from the centerline of each slice to the center of rotation or to the center of moments
e	=	The vertical distance from the centroid of each slice to the center of rotation or to the center moments
d	=	The perpendicular distance from a line load to the center of rotation or to the center of moments
ω	=	The angle of the line load from the horizontal
α	=	The angle between the tangent to the center of the base of each slice and the horizontal

Figure A1- Forces Acting on a Slice through a Sliding Mass Defined by a Fully Specified Slip Surface (after Fredlund et al, 1981)

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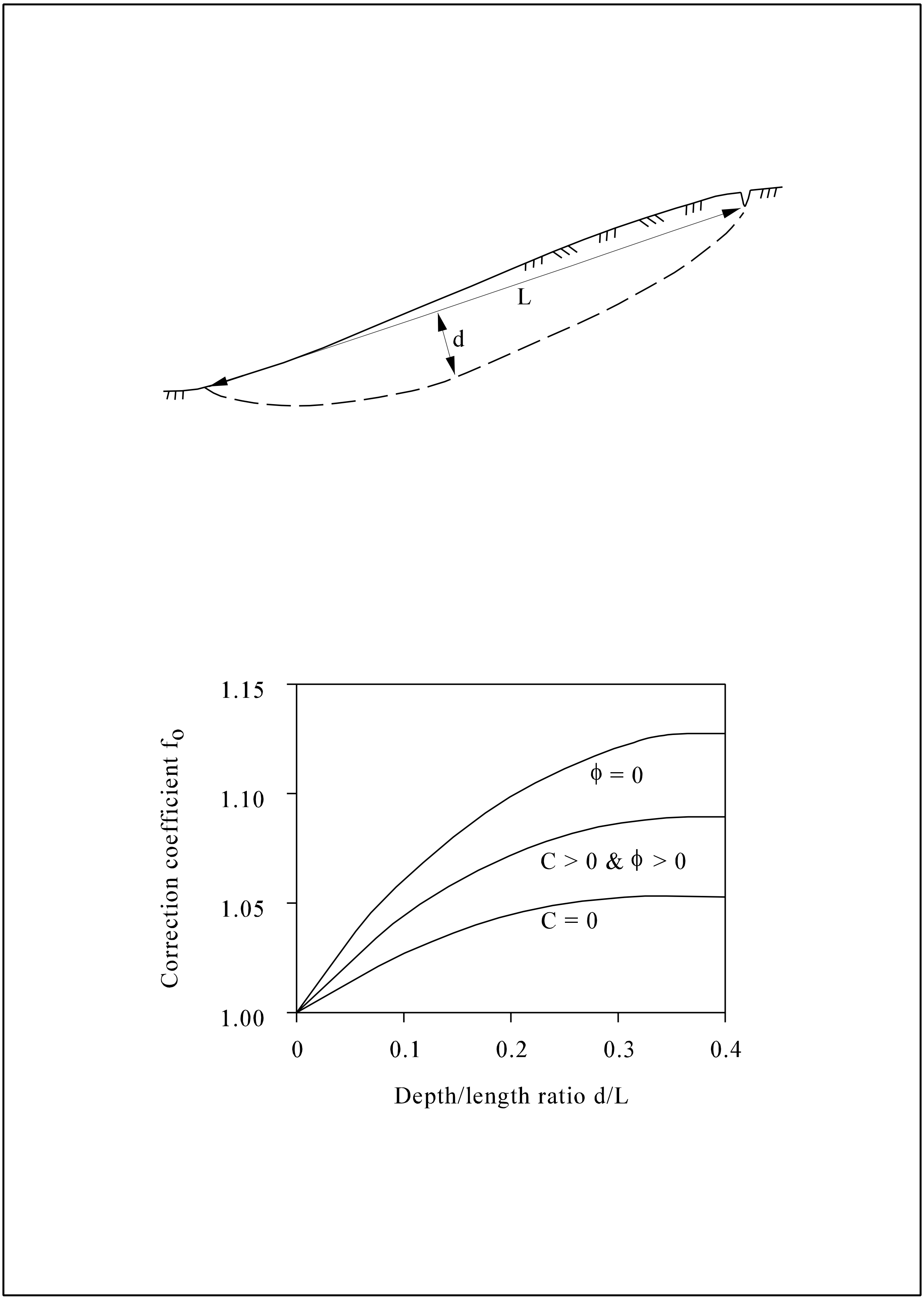

Figure A2 - Correction Factor f_o for Janbu's Simplified Method
(after Janbu et al, 1973)

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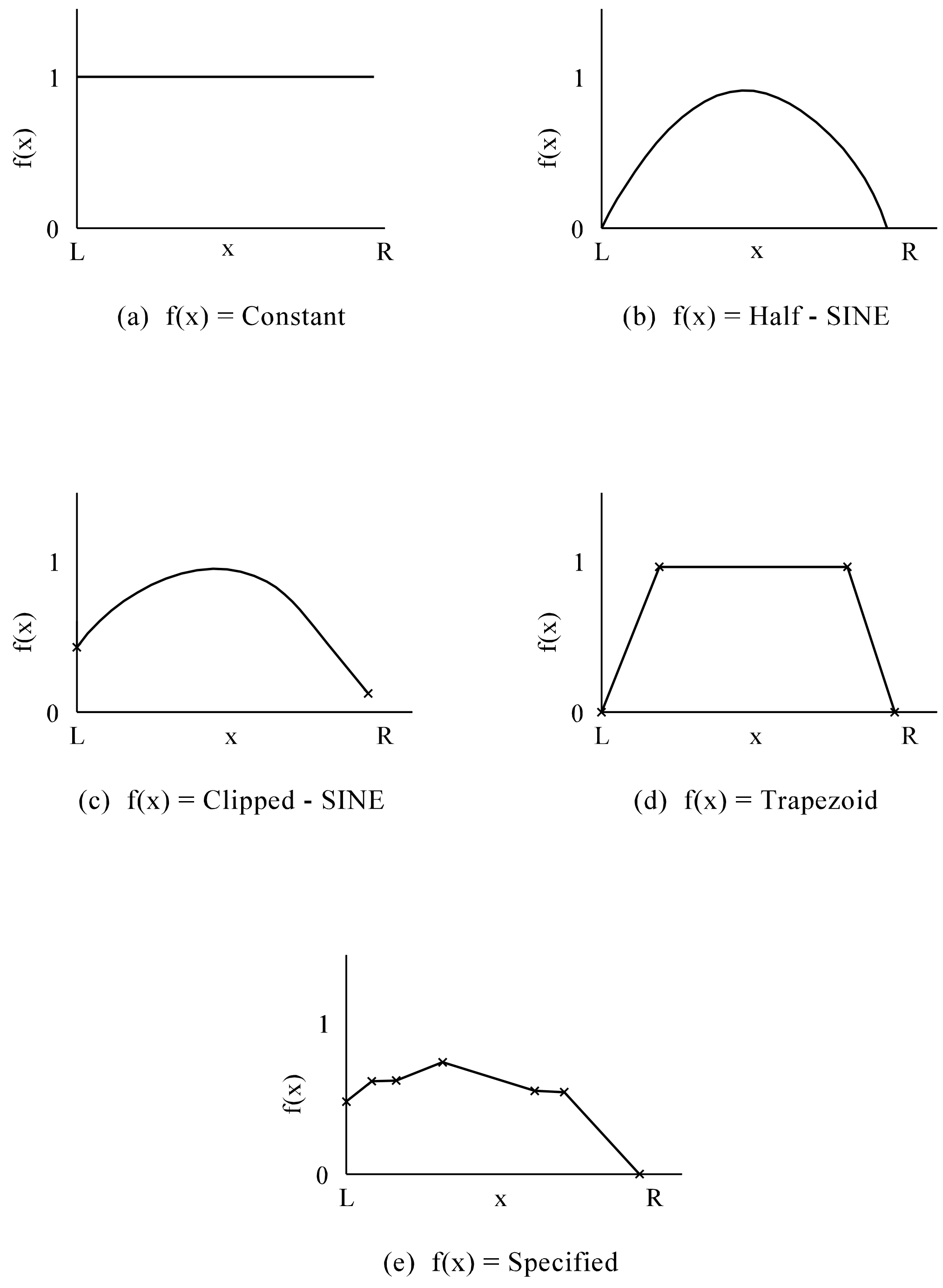

Legend:

L	Left	R	Right

Figure A3 - Typical Functional Variations for the Direction of the Interslice Force with Respect to the x Direction (after Fredlund et al, 1981)

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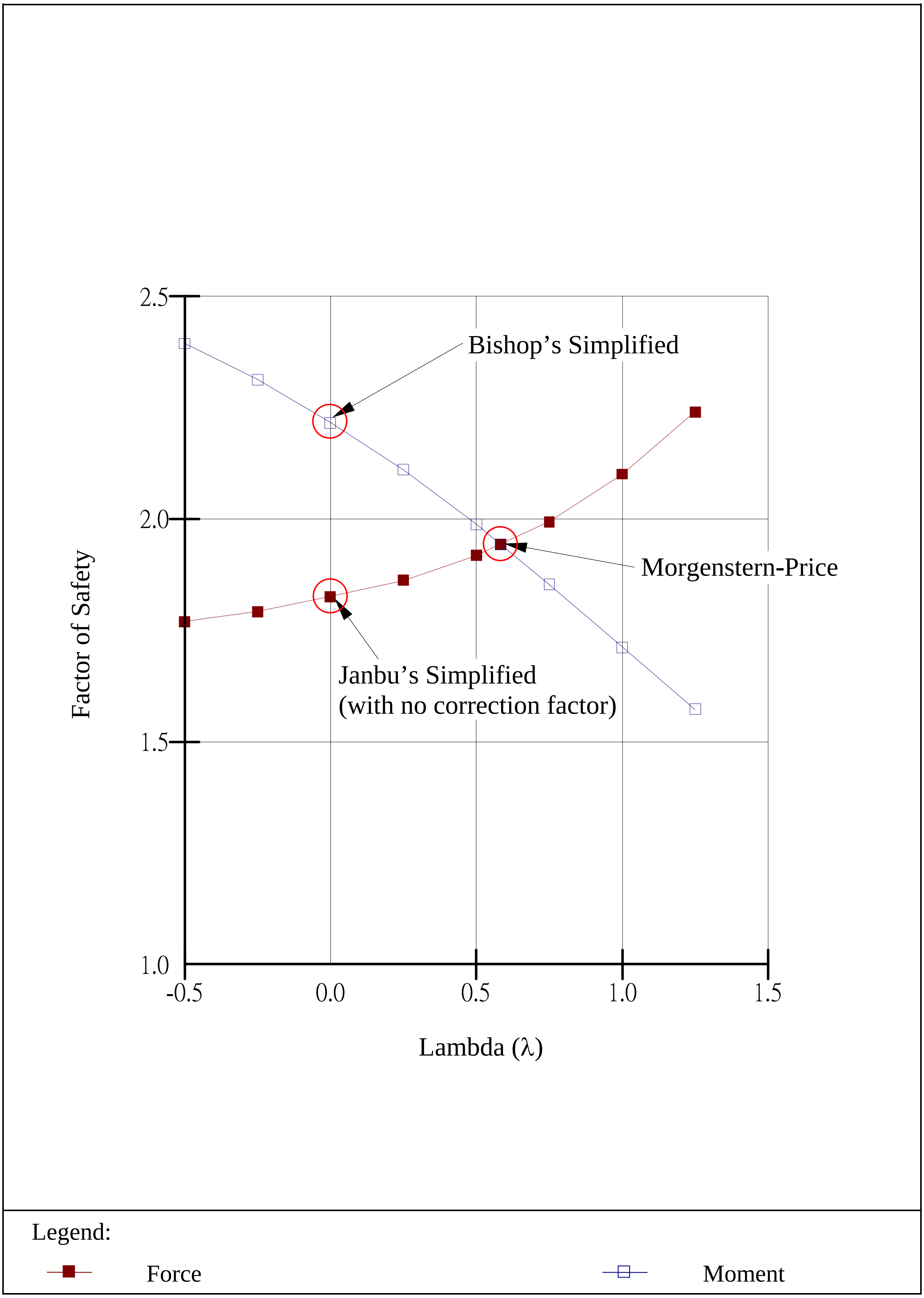

Figure A4 - Typical Plot of Factor of Safety (FoS) versus Lambda (λ) Using the GLE Formulation

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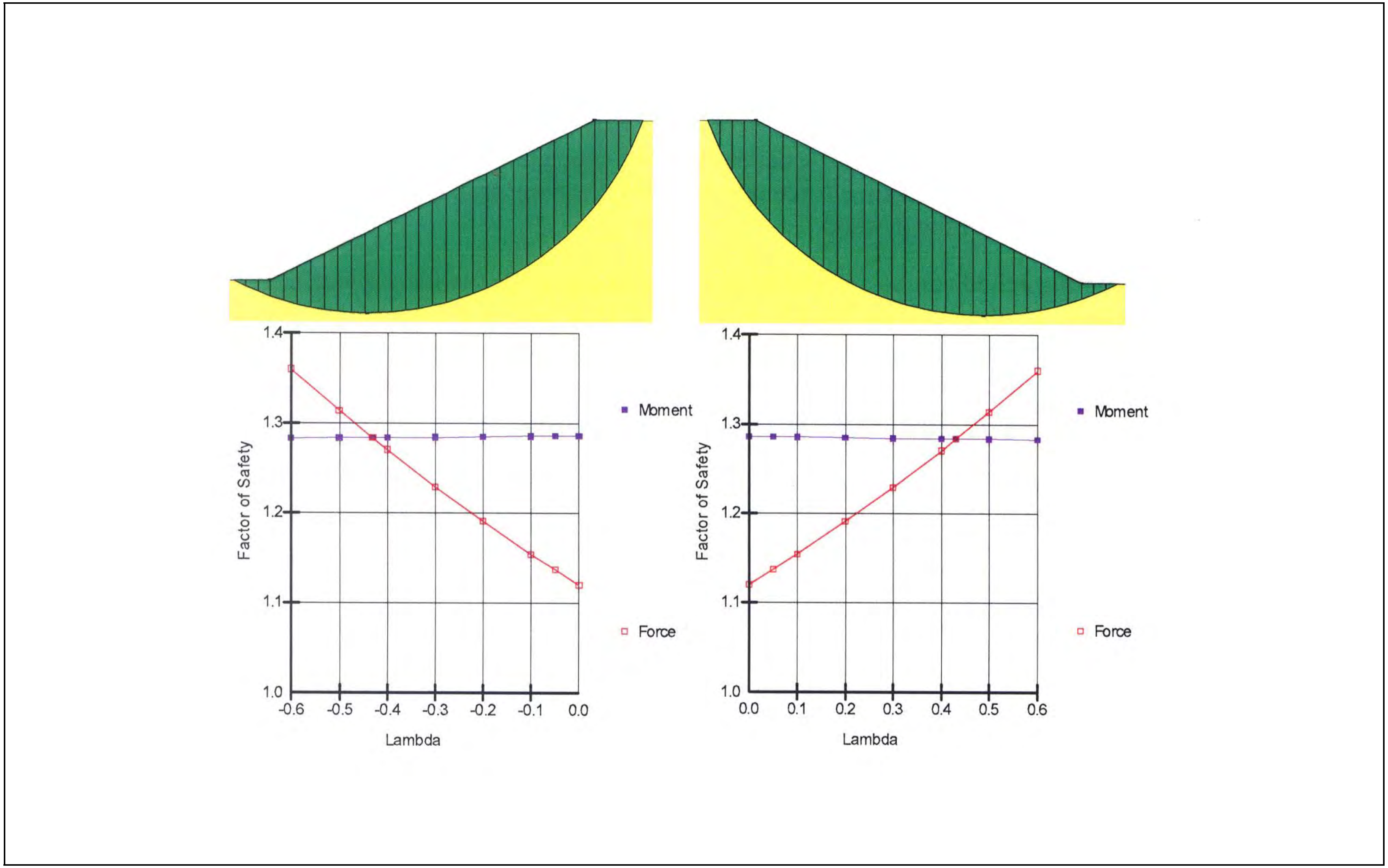

Figure A5 - Effect of Sliding Direction on Lambda (λ) (Krahn, 2004)

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APPENDIX B

NAILED EXCAVATION - EFFECT OF LOCATION OF APPLIED NAIL FORCES

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B.1 NAILED EXCAVATION

B.1.1 Determination of Axial Nail Forces

The vertical excavation was taken to be 6 m in height (see Figure 16). Four rows of 8 m long nails, each with a 40 mm diameter bar in a 100 mm diameter grouted hole, were installed at an inclination of 10° below the horizontal to maintain the stability of the excavation during construction. Both the vertical and horizontal spacings were taken as 1.5 m. The soil parameters were c' = 5 kPa and ϕ' = 39°, unit weight = 19 kN/m^3 and an elastic modulus = 10 MPa. The shotcrete facing used was 100 mm in thickness and it was assumed to be connected to the soil nails.

The axial forces mobilised in the nails were determined using the two-dimensional finite difference code, Fast Lagrangian Analysis of Continua (FLAC). The construction sequence of the nailed excavation was modelled in the analysis (see Figure B1). Construction progressed incrementally in a top down manner by repeating two steps of construction. The first step began with soil being excavated to a depth of 0.5 m below the level of soil nail. Step 2 consisted of installing the soil nail and concrete facing. Steps 1 and 2 were repeated until the full excavation depth (6 m) was attained.

From the FLAC results, the axial forces developed in the nails at the final stage of excavation are given below:

Soil Nail Number	Axial Nail Force, T
N1	60 kN/m
N2	40 kN/m
N3	25 kN/m
N4	11 kN/m
	ΣT = 136 kN/m

The above nail forces are used to compare the different limit equilibrium methods of slices in this study.

B.2 LOADING CONDITIONS

To investigate the effects of the location of applied nail forces, five loading conditions (Figure B2) are considered in the analysis:

(a) Different axial nail forces (T) are applied at individual nail locations;

(b) The total of the axial nail forces (ΣT) is distributed equally among the four rows of nails;

(c) The total of the axial forces of the nails is represented by a single force (ΣT) which is applied at the top nail location (N4);

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(d) The single force (ΣT) is applied at the mid-height of the excavation; and

(e) The single force (ΣT) is applied at the bottom nail location (N1).

In each loading condition, the resultant nail force used is the same, i.e. 136 kN/m (ΣT). The nail forces are applied on the face of the excavation for the five loading conditions.

Eleven slip surfaces (S1 to S11) through the retained soil mass behind the shotcrete facing are considered in the analysis, see Figure 17. The FoS for each of slip surfaces is calculated using the four limit equilibrium methods, namely the Janbu's Simplified (JS) method (with and with no correction factor), the Janbu's Generalized (JG) method, the Bishop's Simplified (BS) method and the Morgenstern-Price (M-P) method. The computer software package SLOPE/W 2004 developed by GEO-Slope has been adopted in the analysis.

In the M-P method, the function f(x) is assumed to be constant. Results of comparative analysis has shown that the constant function yields FoS values almost the same as the half-sine function.

The results of the analysis are presented in the following sections.

B.3 RESULT OF ANALYSIS

B.3.1 Loading Conditions (a) – Different Forces Applied at Individual Nail Locations

It should be noted that the JG method always encounters problems of non-convergence and as such, no solutions can be obtained for any of the slips analysed. The factor of safety values computed using the remaining three methods of analysis for the loading condition (a) are plotted in Figure B3. It can be observed that the JS method can produce converged solutions only for slip surfaces S7 to S11, the M-P method for slip surfaces S6 to S11, and the BS method for slip surfaces S4 to S11. For those slips where FoS are computed, the JS method gives the highest FoS and this is followed by the M-P method. The BS method gives the lowest FoS. Despite the disparity, the difference in FoS among the methods is smaller than 6% in respect to the M-P method for all slips. The slips with minimum FoS are at slip surfaces S5 and S7 respectively for the BS and the M-P method. It is unable to determine the minimum FoS for the JS method due to problems of convergence encountered in shallow and steep slips.

Figure B4 shows the FoS versus λ plots for slip surface S7 for loading condition (a).

B.3.2 Loading Conditions (b) – Total Force (ΣT) Distributed Equally among All Nails

It should be noted that the JG method always encounters problems of non-convergence and as such, no solutions can be obtained for any of the slips analysed. The factor of safety values computed using the other three methods of analysis for the loading condition (b) are plotted in Figure B5. It can be observed that the JS method can produce converged solutions only for slip surfaces S7 to S11, the M-P method for slip surfaces S6 to S11, and the BS

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method for slip surfaces S4 to S11. For those slips where FoS are computed, the JS method gives the highest FoS and this is followed by the M-P method. The BS method usually gives the lowest FoS. The differences in FoS among the methods are about -7% for the JS method and 13% for the BS method in respect to the M-P method for all converged slips. The slips with minimum FoS are at slip surfaces S5 and S7 respectively for the BS and the M-P method. It is unable to determine the minimum FoS for the JS method due to problems of convergence encountered in shallow and steep slips.

Figure B4 shows the FoS versus λ plots for slip surface S7 for loading condition (b).

B.3.3 Loading Condition (c) - Single Force (ΣT) Applied at Location of Top Nail (N4)

It should be noted that the JG method always encounters problems of non-convergence and as such, no solutions can be obtained for any of the slips analysed. Figure B6 compares the FoS obtained for the slip surfaces using the other three methods of analysis for loading condition (c). It can be observed that both the M-P method and the JS method can produce converged solutions only for slip surfaces S7 to S11, and the BS method for slip surfaces S4 to S11. For those slips where FoS are computed, the JS method gives the highest FoS and this is followed by the M-P method. The BS method gives the lowest FoS. The difference in FoS among the methods are about 17% for the JS method and -27% for the BS method in respect to the M-P method for slip S7. The slip with minimum FoS is at slip surface S5 for the BS method. It is unable to determine the minimum FoS for the JS and M-P methods due to problems of convergence encountered in shallow and steep slips.

For illustrative purpose, the FoS versus λ plot for slip S7 is presented in Figure B4.

B.3.4 Loading Condition (d) - Single Force (ΣT) Applied at Mid-height of the Excavation

It should be noted that the JG method always encounters problems of non-convergence and as such, no solutions can be obtained for any of the slips analysed. Figure B7 shows the FoS for the slip surfaces calculated by the other three methods for loading condition (d). It can be observed that the JS method can produce converged solutions only for slip surfaces S7 to S11, the M-P method for slip surfaces S6 to S11, and the BS method for slip surfaces S4 to S11. The shapes of the curves are very similar to those in condition (c), but the difference in FoS for each method is smaller. For those slips where FoS are computed, the JS method gives the highest FoS and this is followed by the M-P method. The BS method gives the lowest FoS. It is unable to determine the minimum FoS for the JS and M-P methods due to problems of convergence encountered in shallow and steep slips.

For illustrative purpose, the FoS versus λ plot for slip S7 is presented in Figure B4.

B.3.5 Loading Condition (e) - Single Force (ΣT) Applied at the Bottom Nail Location (N1)

It should be noted that the JG method always encounters problems of non-convergence and as such, no solutions can be obtained for any of the slips analysed. Figure B8 compares the FoS obtained for the slip surfaces using the other three methods for loading condition (e).

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It can be observed that the JS method can produce converged solutions for slip surfaces S7 to S11, the M-P method for slip surfaces S6 to S11, and the BS method for slip surfaces S4 to S11. The differences in FoS among the methods are small. For those slips where FoS can be computed, the BS method gives the highest FoS and this is followed by the M-P method. The BS method gives the lowest FoS. This shows a reversal of relative magnitudes of the minimum FoS as computed in loading conditions (c) and (d).

For illustrative purpose, the FoS versus λ plot for slip S7 is presented in Figure B4.

B.3.6 Comparison of Factors of Safety for Loading Conditions (c), (d) and (e)

B.3.6.1 Janbu's Simplified Method

Figure B9 shows the factors of safety computed using the JS method for the slip surfaces for the three loading conditions (c), (d) and (e) for the case where no correction factor is applied and the case where correction factors are applied in the JS method. For the converged slip surfaces, the computed FoS are so close that they are practically identical among the three loading conditions in both cases.

B.3.6.2 Bishop's Simplified Method

The factors of safety calculated using the BS method for the three loading conditions (c), (d) and (e) are plotted in Figure B10. For those converged slip surfaces, loading condition (e) (the single nail force applied at toe) gives the highest FoS. Loading condition (d) gives FoS higher than those for loading condition (c) but lower than those for condition (e).

B.3.6.3 Morgenstern-Price Method

Figure B11 shows the FoS computed using the M-P method for the slip surfaces for the three loading conditions (c), (d) and (e). For those converged slip surfaces, the single nail force applied at the toe (condition (e)) gives the highest FoS. The load condition (d) gives FoS higher than those for condition (c) but lower than those for condition (e). The differences in the values of FoS are about 10% in condition (c) (force at top) and -10% in condition (e) (force at toe) in respect to condition (d) (force at middle) for the all converged slip surfaces.

B.3.6.4 Comparison of method results

Results of the comparative analysis indicate that JS method yields similar factor of safety values for the five loading conditions (a) to (e). On the contrary, the factor of safety values computed using the BS method and the M-P method are sensitive to the location where the resultant nail force is applied. Where the resultant nail load is applied at the lower part of the wall, it gives a higher FoS than that applied at the upper part of the wall. The reason for this is that the locations of the resultant nail force control the magnitudes of resisting moment in the moment equilibrium equation F_m, see Equation (A2) and Figure A1. This

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aspect can be looked at further by examining at the FoS versus λ plots. Figure B4 depicts such plots for slip S7 for the five loading conditions. For ease of illustration, they are combined into one plot as displayed in Figure B12.

B.3.7 Comparison of Values of Minimum Factor of Safety

Problems of convergence are encountered for some of the shallow and steep slips in all the methods. Slips with the minimum FoS can be located using the BS method for all the loading conditions. With the M-P method, the slips with the minimum FoS can be located only for loading conditions (a), (b) and (e). The JS and the JG methods cannot identify the slips with the minimum FoS for any of the loading conditions. Because of this problem, a comparison of values of minimum FoS cannot be made for the various methods.

The results of this study for the nailed excavation illustrate the situation where the minimum FoS of a vertical wall is difficult to obtain using the limit equilibrium methods of slices because of convergence problems encountered on shallow and steep slip surfaces.

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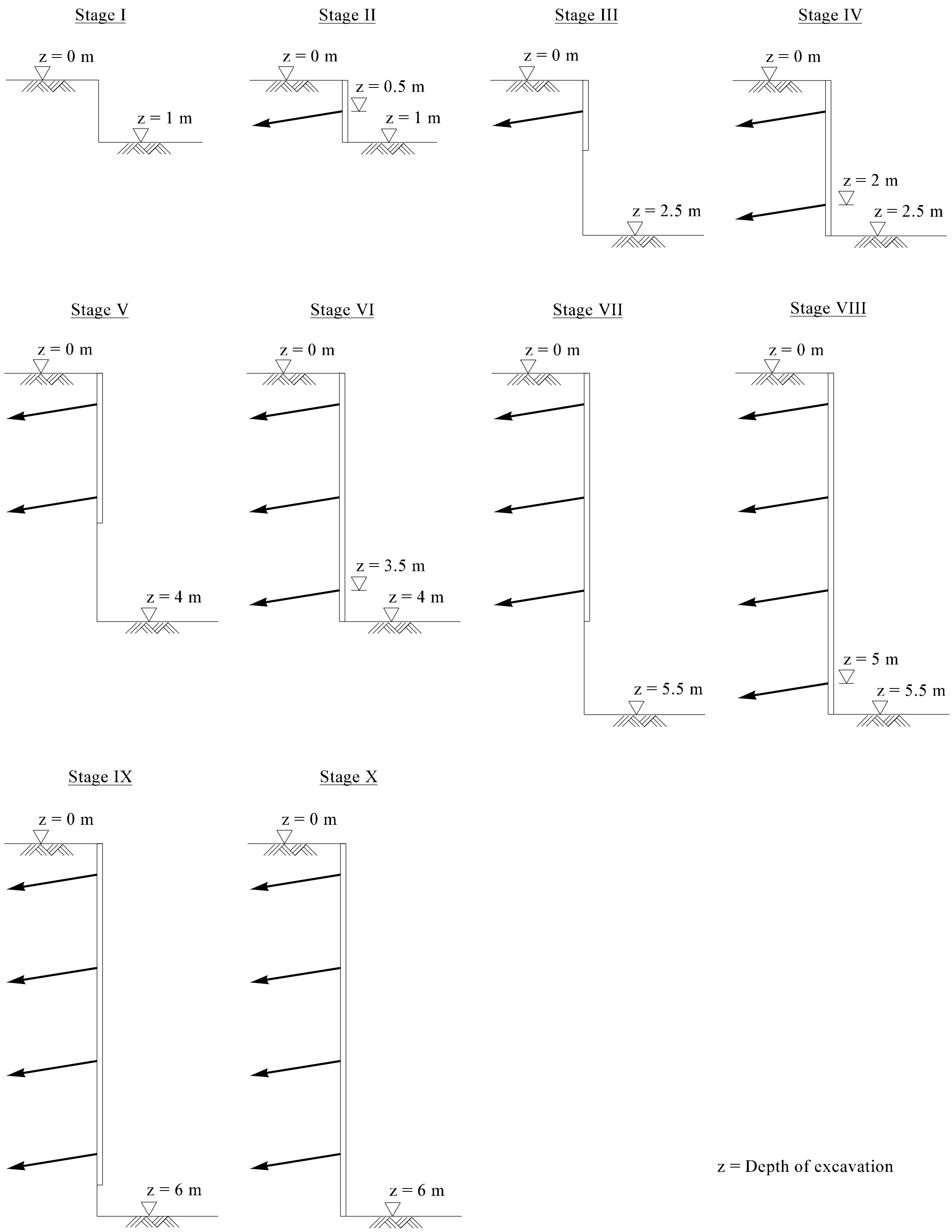

Notes:	Stage I:	Excavate to z = 1 m.	Stage II:	Install soil nail at z = 0.5 m and shotcreting.
	Stage III:	Excavate to z = 2.5 m.	Stage IV:	Install soil nail at z = 2 m and shotcreting.
	Stage V:	Excavate to z = 4 m.	Stage VI:	Install soil nail at z = 3.5 m and shotcreting.
	Stage VII:	Excavate to z = 5.5 m.	Stage VIII:	Install soil nail at z = 5 m and shotcreting.
	Stage IX:	Excavate to z = 6 m.	Stage X:	Shotcreting to full depth.

Figure B1 - Excavation Sequence Simulated in Numerical Analysis

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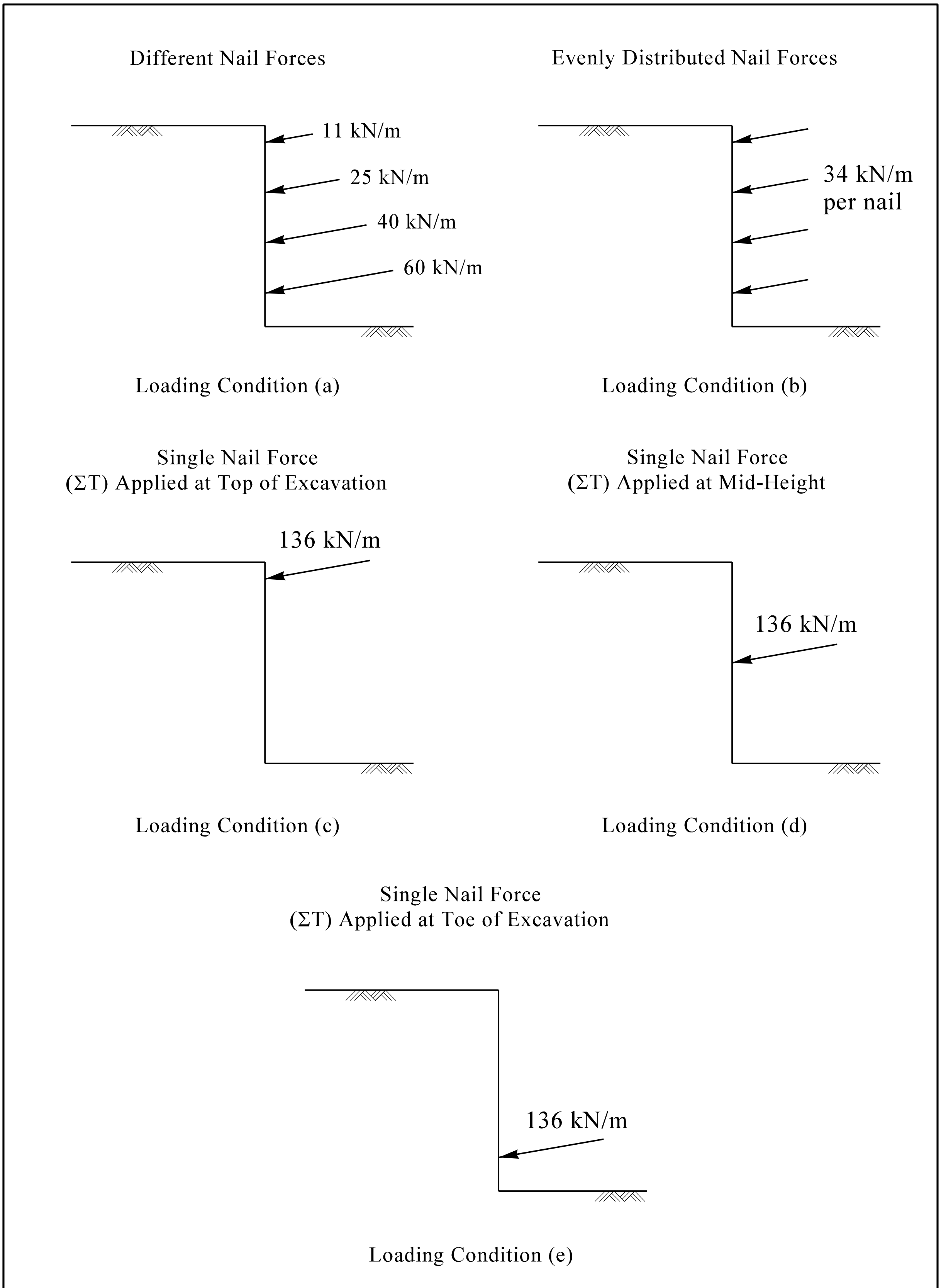

Note: Distribution of nail forces for loading condition (a) is obtained from FLAC analysis.

Figure B2 - Loading Conditions for Nailed Excavation

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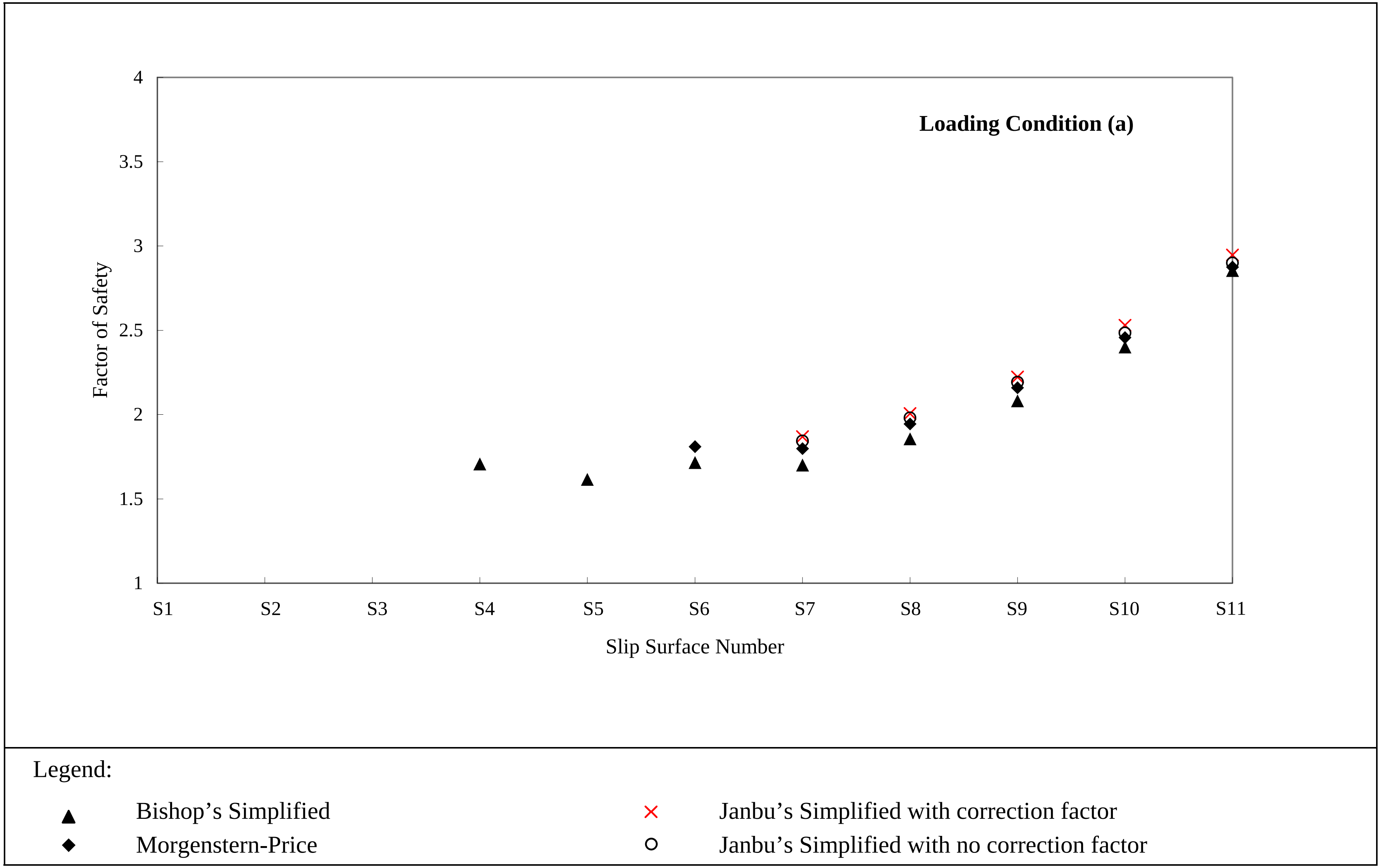

Figure B3 - Factor of Safety versus Slip Surface Number for Loading Condition (a)

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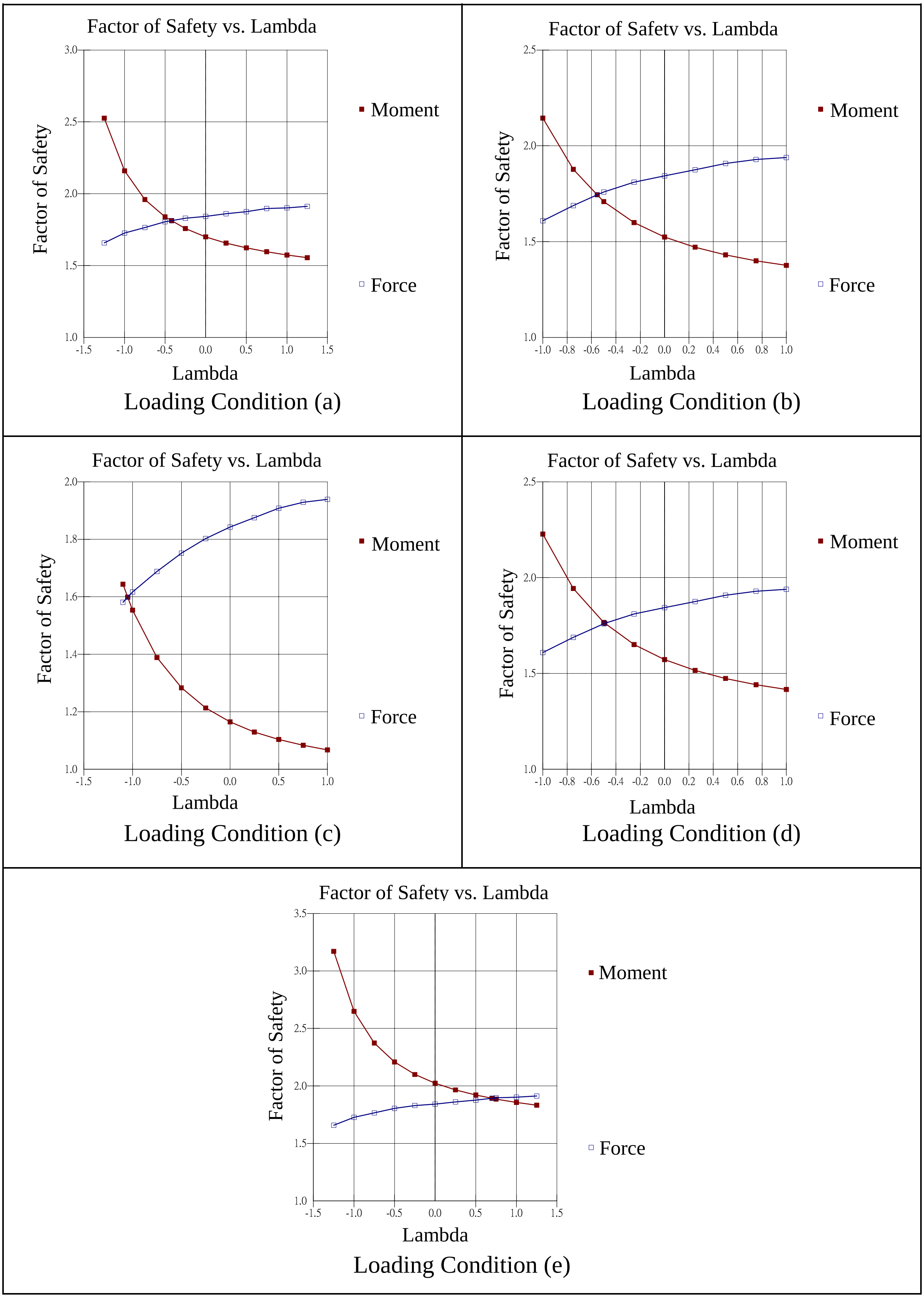

Figure B4 - Factor of Safety (FoS) versus Lambda (λ) for Slip Surface S7 for Loading Conditions (a) to (e) in the Nailed Excavation Model

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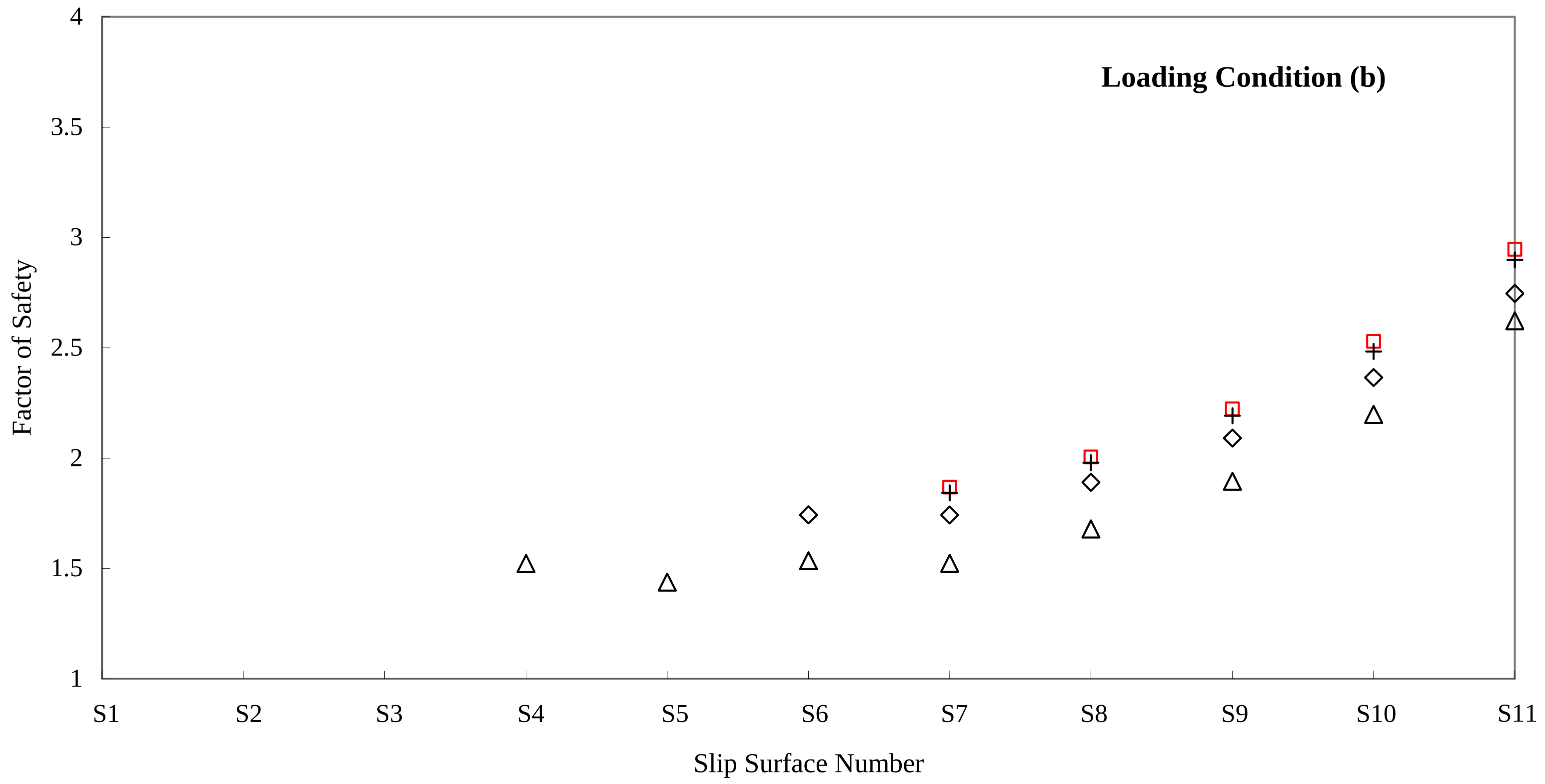

Legend:

△	Bishop's Simplified	□	Janbu's Simplified with correction factor
◇	Morgenstern - Price	+	Janbu's Simplified with no correction factor

Figure B5 - Factor of Safety versus Slip Surface Number for Loading Condition (b)

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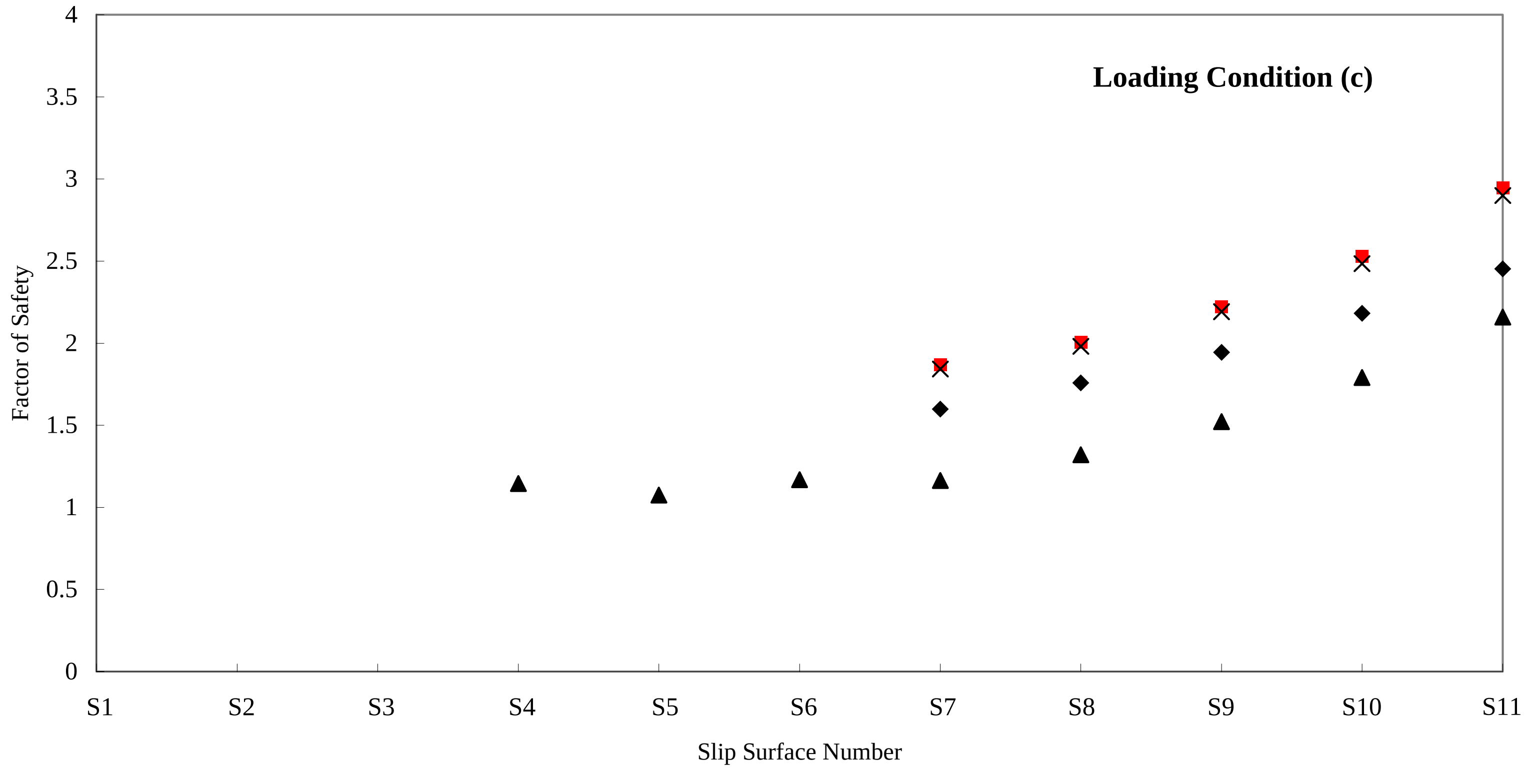

Legend:

▲	Bishop's Simplified	■	Janbu's Simplified with correction factor
◆	Morgenstern-Price	×	Janbu's Simplified with no correction factor

Figure B6 - Factor of Safety versus Slip Surface Number for Loading Condition (c)

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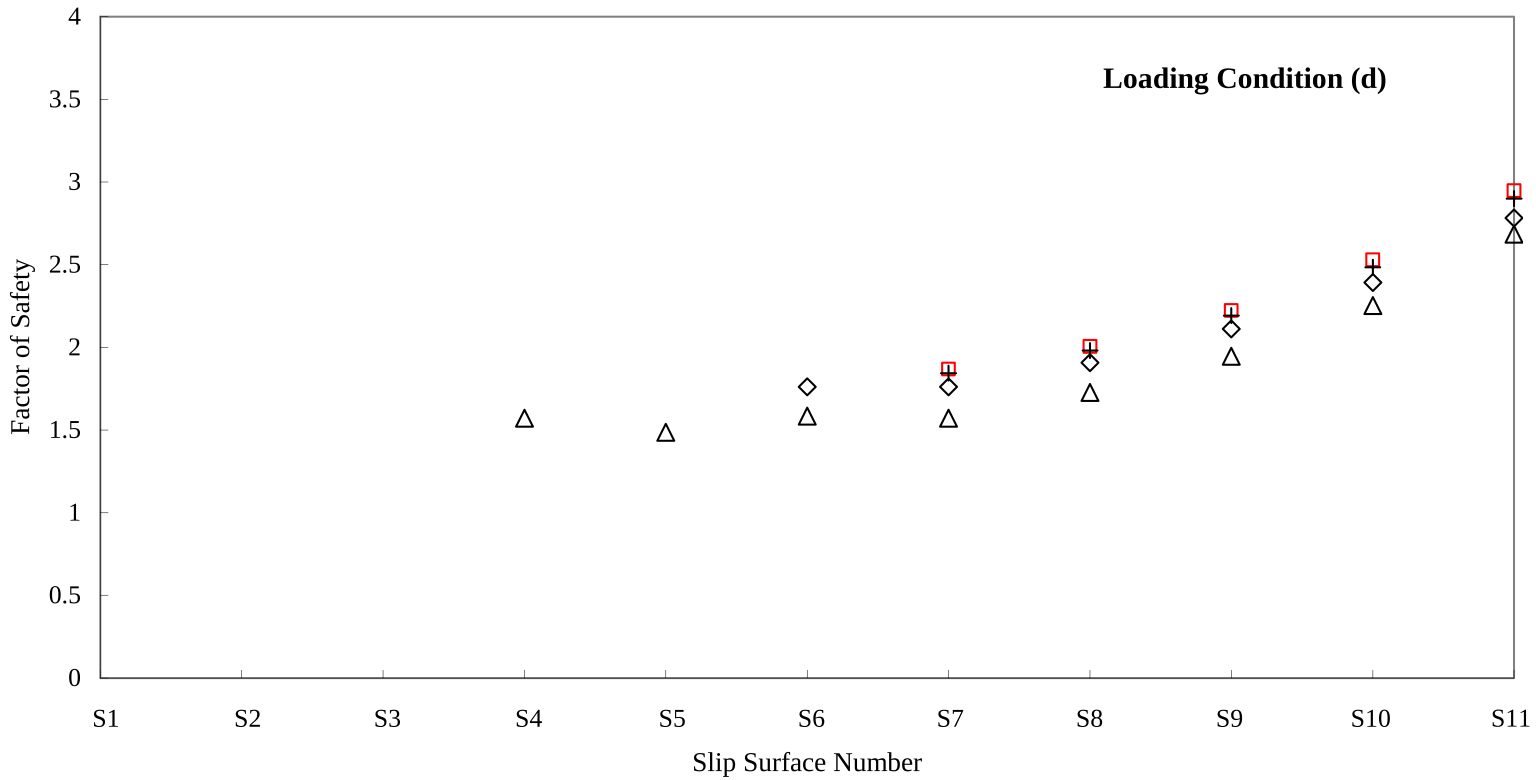

Legend:

△	Bishop's Simplified	□	Janbu's Simplified with correction factor
◇	Morgenstern - Price	+	Janbu's Simplified with no correction factor

Figure B7 - Factor of Safety versus Slip Surface Number for Loading Condition (d)

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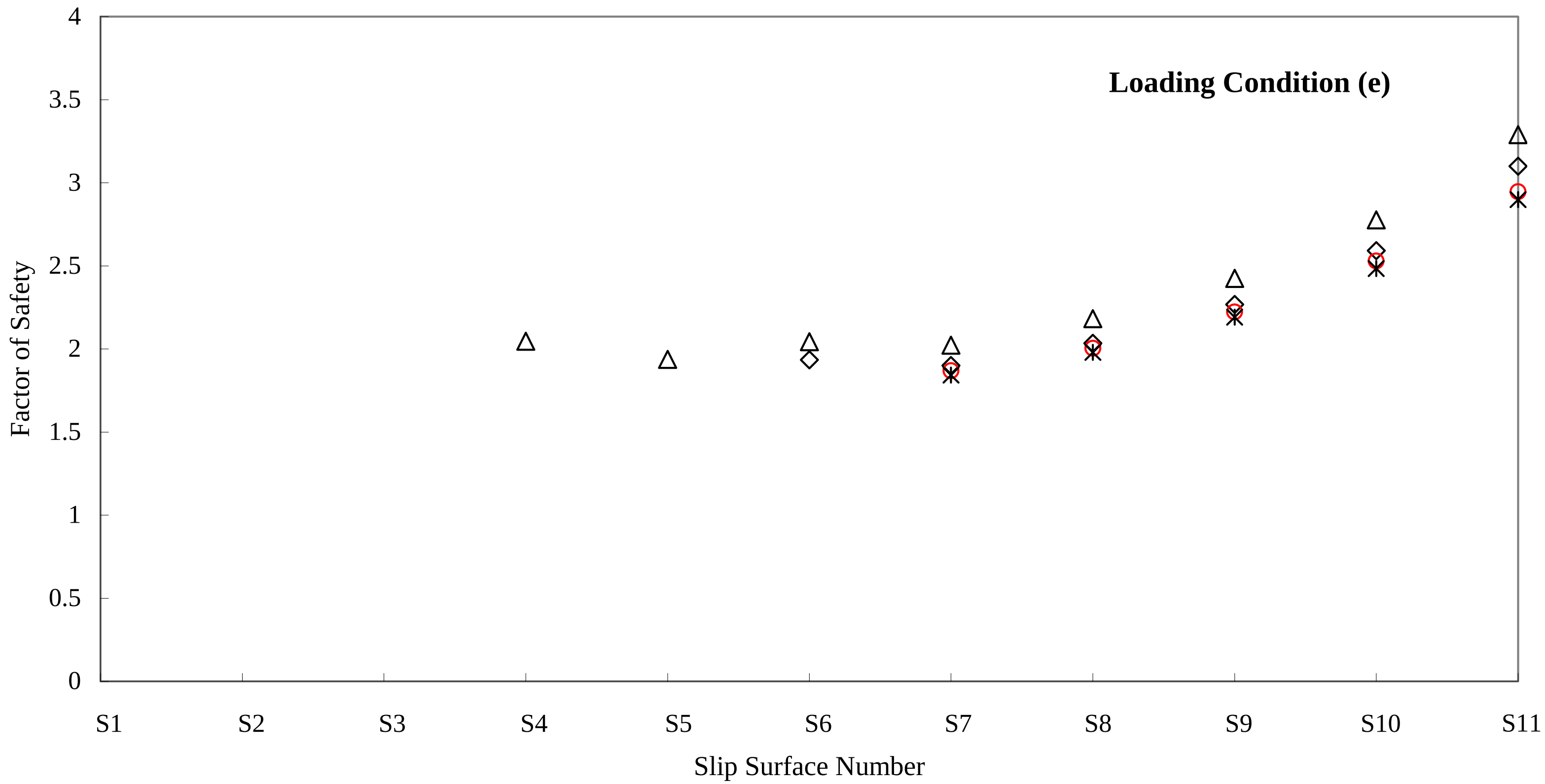

Legend:

△	Bishop's Simplified	○	Janbu's Simplified with correction factor
◇	Morgenstern - Price	✱	Janbu's Simplified with no correction factor

Figure B8 - Factor of Safety versus Slip Surface Number for Loading Condition (e)

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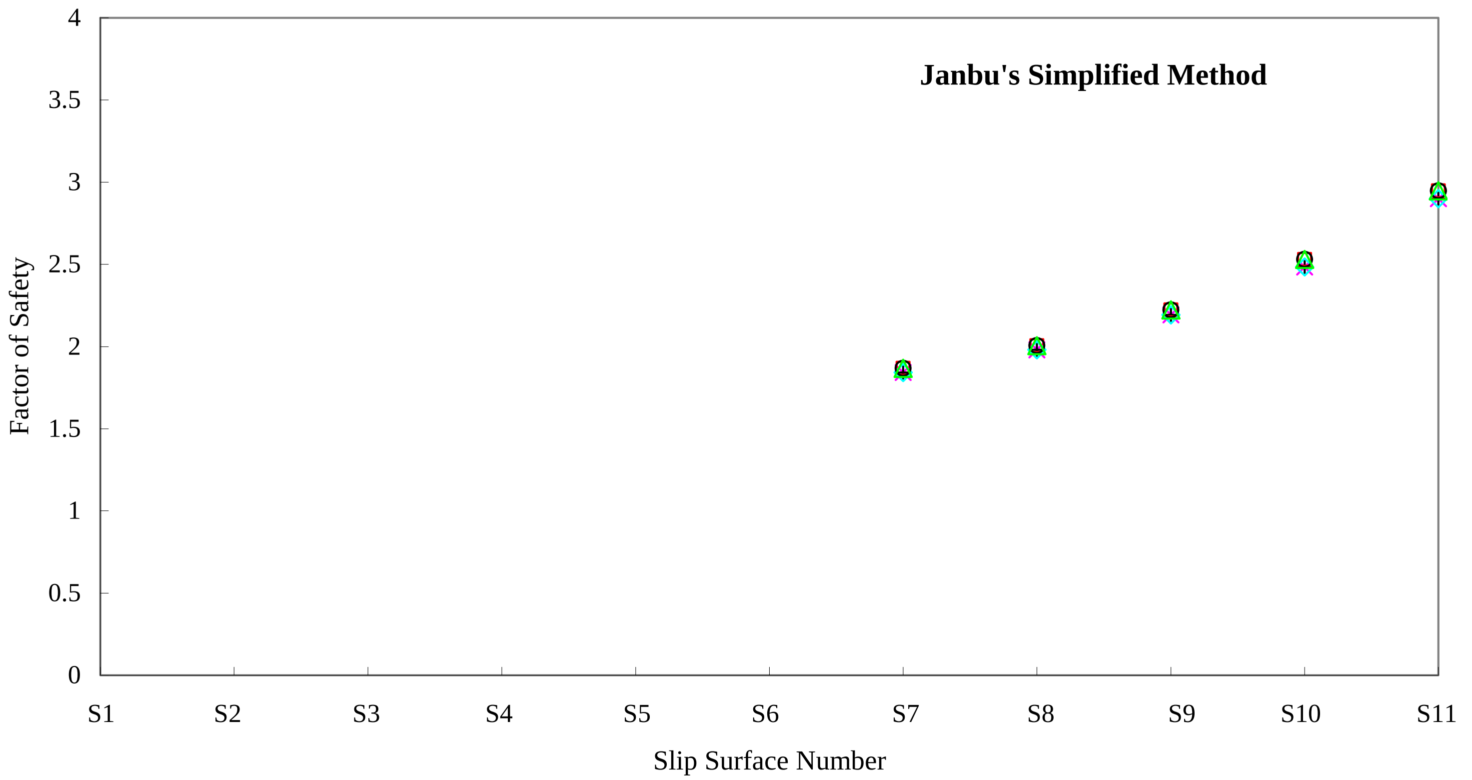

Legend:

△	With correction factor (Loading condition (c))	◇	With no correction factor (Loading condition (c))
□	With correction factor (Loading condition (d))	+	With no correction factor (Loading condition (d))
○	With correction factor (Loading condition (e))	✳	With no correction factor (Loading condition (e))

Figure B9 - Factor of Safety Values Computed Using Janbu’s Simplified Method for Loading Conditions (c), (d) and (e)

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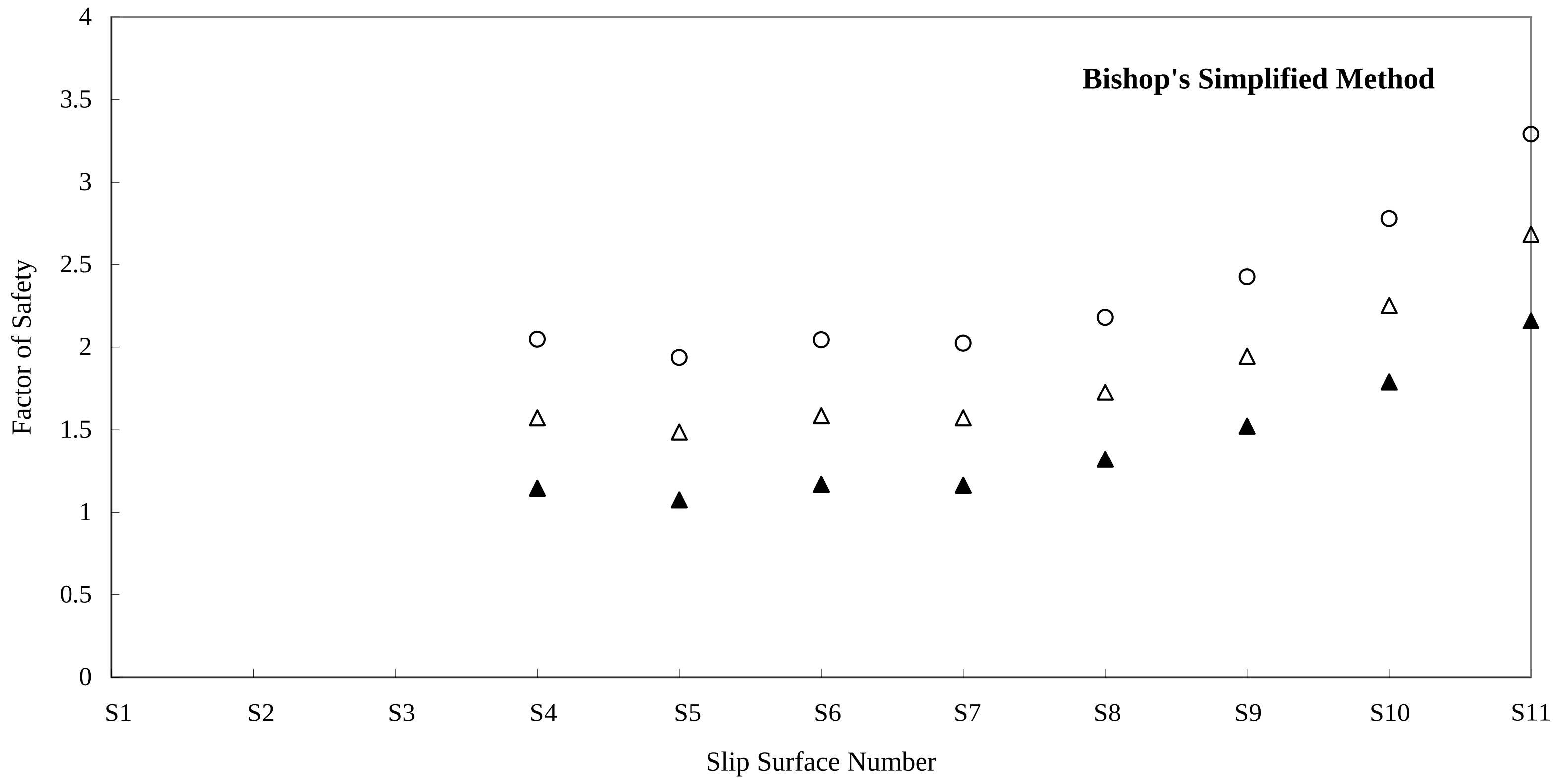

Legend:

▲ Loading Condition (c) △ Loading Condition (d) ○ Loading Condition (e)

Figure B10 - Factor of Safety Values Computed Using Bishop's Simplified Method for Loading Conditions (c), (d) and (e)

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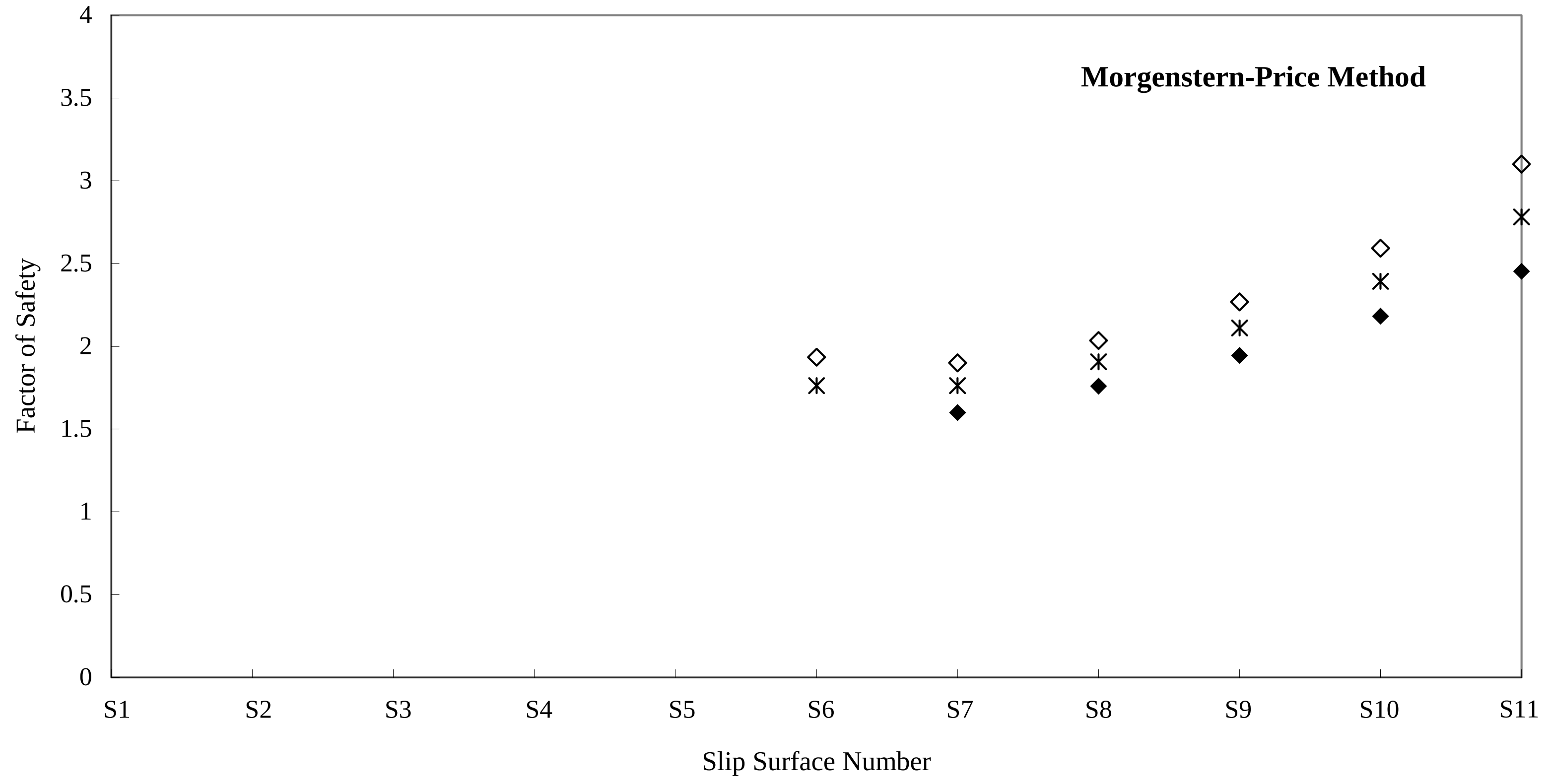

Legend:

◆ Loading Condition (c) ✱ Loading Condition (d) ◇ Loading Condition (e)

Figure B11 - Factor of Safety Values Computed Using Morgenstern-Price Method for Loading Conditions (c), (d) and (e)

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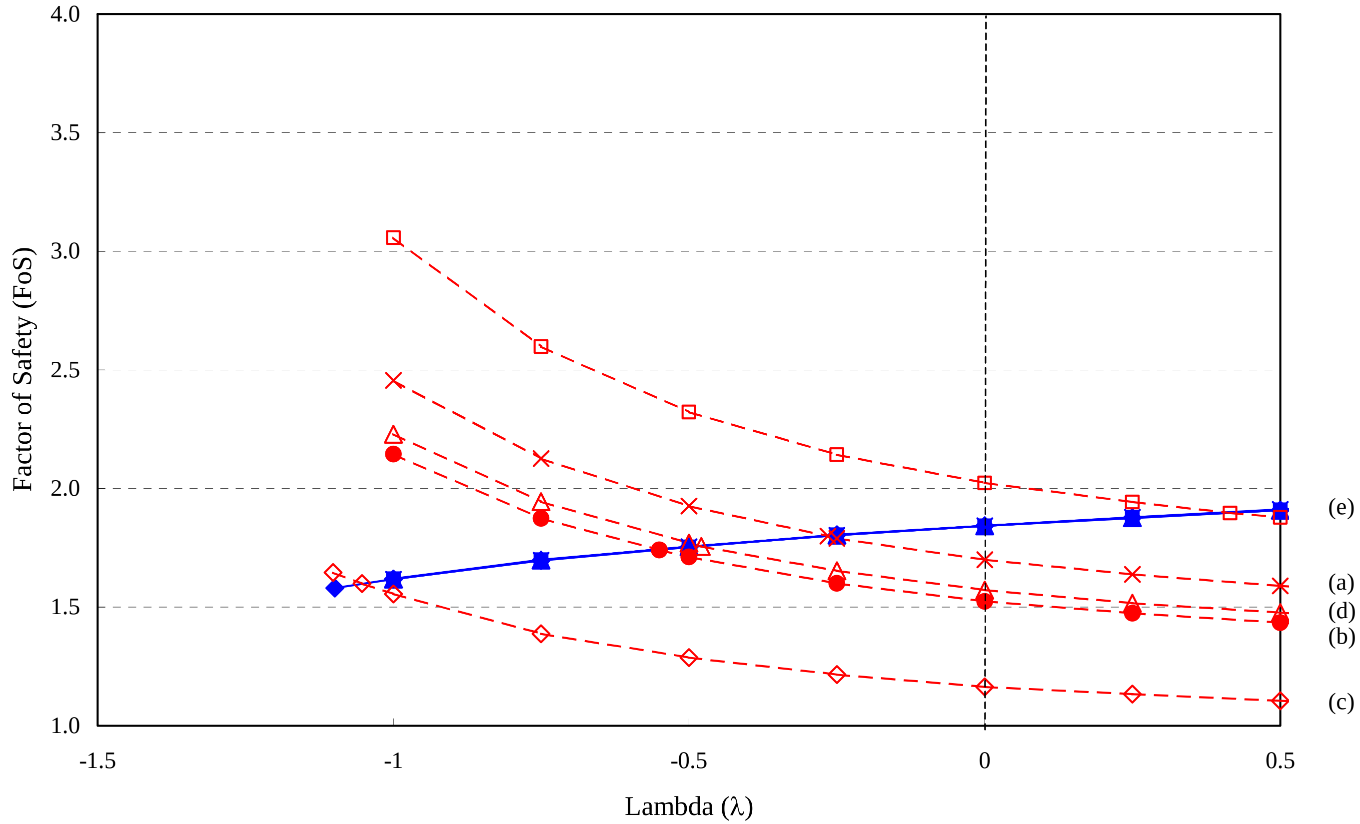

Legend:

Loading condition (a) - Force	Loading condition (a) - Moment
Loading condition (b) - Force	Loading condition (b) - Moment
Loading condition (c) - Force	Loading condition (c) - Moment
Loading condition (d) - Force	Loading condition (d) - Moment
Loading condition (e) - Force	Loading condition (e) - Moment

Figure B12 - Factor of Safety (FoS) versus Lambda (λ) Plot for Slip Surface S7 for Loading Conditions (a), (b), (c), (d) & (e) for the Nailed Excavation

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APPENDIX C

DERIVATION OF SHEAR STRENGTH CONTRIBUTED BY THE VERTICAL AND THE HORIZONTAL FORCE COMPONENTS IN A SOIL NAIL

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In Figure C1, the contribution of shear strength from the vertical and the horizontal force components of a nail force is considered using a frictional model (c' = 0). The contribution of shear resistance S for a soil nail subject to an axial force F and with an inclination angle ω with the horizontal, is given by:

$$S = F \cos\theta \tan\phi' + F \sin\theta \quad \text{..(C1)}$$

where θ is the angle made between the normal of the slip plane and the soil nail at the point where they intersect, and ϕ' is the effective friction angle of the soil.

The contribution of shear resistance (S_h) along the slip surface due to the horizontal component of F is:

$$S_h = F \cos\omega\, [\cos(\omega+\theta) \tan\phi' + \sin(\omega+\theta)] \quad \text{................................(C2)}$$

Similarly, the contribution of shear resistance (S_v) from the vertical component of F is:

$$S_v = F \sin\omega\, [\sin(\omega+\theta) \tan\phi' - \cos(\omega+\theta)] \quad \text{................................(C3)}$$

It can be shown that $S = S_h + S_v$ from the above equations. Results of S, S_h and S_v for ω = 10°, 20° and 30°, θ = 10°, 20°, 30° and 40°, ϕ' = 35° and 40° are tabulated in Table C1.

It can be noted from Table C1 that S_v can be positive, zero or negative, contributing a positive, null or negative effect on the shear resistance S respectively, depending on the following relationships:

$$S_v > 0 \quad \text{if } \phi' > \pi/2 - \omega - \theta$$
$$S_v = 0 \quad \text{if } \phi' = \pi/2 - \omega - \theta$$
$$S_v < 0 \quad \text{if } \phi' < \pi/2 - \omega - \theta$$

Also it is shown in Equation (C3) that S_v generally increases with the nail inclination ω. When S_v is positive, S_h is smaller than S and it represents a stabilizing effect from the vertical component. When S_v is negative, S_h is larger than S, indicating a de-stabilizing effect from the vertical component.

The effect of S_v is small when ω is small. Since soil nails are normally installed at small inclination (i.e. small ω), ignoring S_v will not give significant difference in results.

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Table C1 - Results of Comparative Analysis Using Frictional Model (c' = 0)

$\omega = 10^o$ $\phi' = 35^o$

θ (Degrees)	S_v	S_h	S	S_v+S_h-S	S_h/S
10	-0.122	0.985	0.863	0.000	1.141
20	-0.090	1.090	1.000	0.000	1.090
30	-0.055	1.161	1.106	0.000	1.050
40	-0.018	1.198	1.179	0.000	1.016

$\omega = 10^o$ $\phi' = 40^o$

θ (Degrees)	S_v	S_h	S	S_v+S_h-S	S_h/S
10	-0.113	1.113	1.000	0.000	1.113
20	-0.078	1.208	1.131	0.000	1.069
30	-0.039	1.266	1.227	0.000	1.032
40	0.000	1.286	1.286	0.000	1.000

$\omega = 20^o$ $\phi' = 35^o$

θ (Degrees)	S_v	S_h	S	S_v+S_h-S	S_h/S
10	-0.176	1.040	0.863	0.000	1.204
20	-0.108	1.108	1.000	0.000	1.108
30	-0.036	1.143	1.106	0.000	1.033
40	0.036	1.143	1.179	0.000	0.969

$\omega = 20^o$ $\phi' = 40^o$

θ (Degrees)	S_v	S_h	S	S_v+S_h-S	S_h/S
10	-0.153	1.153	1.000	0.000	1.153
20	-0.078	1.208	1.131	0.000	1.069
30	0.000	1.227	1.227	0.000	1.000
40	0.078	1.208	1.286	0.000	0.940

$\omega = 30^o$ $\phi' = 35^o$

θ (Degrees)	S_v	S_h	S	S_v+S_h-S	S_h/S
10	-0.158	1.021	0.863	0.000	1.183
20	-0.053	1.053	1.000	0.000	1.053
30	0.053	1.053	1.106	0.000	0.952
40	0.158	1.021	1.179	0.000	0.866

$\omega = 30^o$ $\phi' = 40^o$

θ (Degrees)	S_v	S_h	S	S_v+S_h-S	S_h/S
10	-0.113	1.113	1.000	0.000	1.113
20	0.000	1.131	1.131	0.000	1.000
30	0.113	1.113	1.227	0.000	0.908
40	0.223	1.062	1.286	0.000	0.826

Legend:

S_v Shear resistance contributed by the vertical component of nail force (in terms of F in Figure C1)

S_h Shear resistance contributed by the horizontal component of nail force (in terms of F in Figure C1)

S Shear resistance contributed by nail force (in terms of F in Figure C1)

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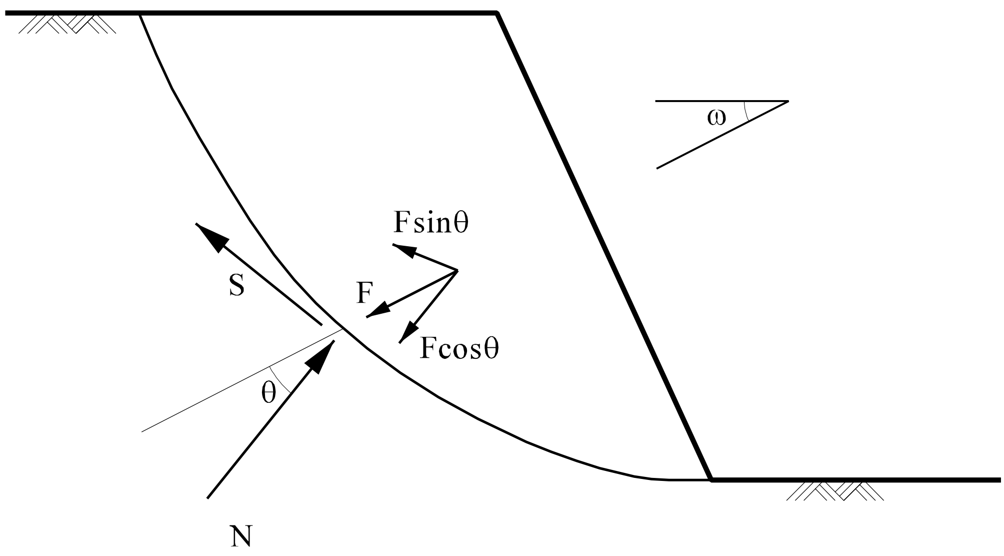

Legend:

S	Shear resistance along slip surface
N	Normal Force
F	Axial force of soil nail
θ	Angle between soil nail and the normal to the slip surface
Fsinθ	Tangential component of F
Fcosθ	Normal component of F
ω	Inclination of the soil nail below the horizontal

Figure C1 - Derivation of Shear Contribution from Nail Force Components Using a Friction Model (c' = 0)

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APPENDIX D

OPTIMIZATION USING SIMULATED ANNEALING TECHNIQUE

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Simulated annealing is a technique suitable for solving optimization problem in particular when a global extremum is hidden among many local extrema. Annealing is a process that liquids freeze and crystallize, or metals cool and anneal. If the liquid is cooled slowly, the atoms are able to arrange themselves to a state of minimum energy (i.e. to form a crystal). In other words, in a slowly cooling system, the Nature is able to minimize its energy. Simulated annealing algorithm mimics this natural annealing process. This algorithm was first introduced in optimization problems by Kirkpatrick et al in 1983. The basic optimization algorithm is based on the following Boltmann probability equation:

$$P(\Delta \mathrm{E}) = \mathrm{e}^{-\frac{\Delta \mathrm{E}}{\mathrm{kT}}} \qquad \text{(D1)}$$

where $P(\Delta E)$ is the probability of an increase in energy ΔE
T is the temperature where the system is in thermal equilibrium
k is the Boltzmann's constant

In this algorithm if $\Delta E < 0$ (i.e. a move from high energy state to low energy state), the probability will be assigned to unity (i.e. the system always take such a preferable move). On the other hand, if $\Delta E \geq 0$, there will be a tiny probability that the system takes such a worse move. As the temperature T decreases, the probability of a worse move for a given value of ΔE decreases correspondingly. Indeed, the energy state E is the objective function, which is also similar to the factor of safety in slope stability analysis. By adopting this algorithm and following the step below, a slip with minimum factor of safety can be identified.

Step 1: Start with an arbitrary slip and calculate the corresponding factor of safety (FoS)

Step 2: Change the location of slip and calculate the new FoS

Step 3: If the change in FoS is negative (i.e. $\Delta FoS < 0$), accept the new location of slip

Step 4: If $\Delta FoS \geq 0$, calculate the probability for such a change based on the Boltmann probability equation (i.e. $P(\Delta FoS)$)

Step 5: Generate a random number, R, ranges from 0 to 1

Step 6: If $P(\Delta FoS) \geq R$, accept the new location of slip, otherwise start with the old slip

Step 7: Reduce the system temperature T according to a prescribed cooling schedule (e.g. $T_{i+1} = 0.9T_i$)

Step 8: Repeat the searching steps 2 to 7 until a minimum FoS is found.

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APPENDIX E

LIMITATIONS OF LIMIT EQUILIBRIUM METHOD OF SLICES

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According to Wright et al (1973), these methods have several fundamental shortcomings, which include:

(a) Arbitrary assumptions are made so that the normal stress on the shear surface may be determined using only the conditions of static equilibrium. These arbitrary assumptions most frequently concern the locations or directions of side forces on slices.

(b) The methods assume that the factor of safety is the same at all points along the potential slip surface. This is reasonable only at failure, i.e. when the factor of safety equals to 1. When the factor of safety is taken to be the same at all points along the potential slip surface even when the factor of safety is greater than unity, the methods cannot model the some mechanisms such as progressive failure.

(c) The stress-strain characteristics of the soil are ignored and as such deformations with a slope are not simulated.

(d) Some of the equilibrium methods do not satisfy all the conditions of equilibrium.

These shortcomings will mean that inter-slice and slip surface forces may not be representative of the actual in situ ground conditions. They are simply the forces that satisfy assumptions employed in (a) and (b) above for each slice. Tavenas et al (1980) presented cases where the actual ground stresses are substantially different from those assumed in the limit equilibrium methods. Krahn (2003) pointed out that if the inter-slice forces are not representative of actual ground conditions, it is not possible to determine a realistic line of thrust for the inter-slice shear-normal resultant and as such, the forces on each slice can result in a line of thrust outside the slice. This indicates that the inter-slice forces are not always realistic in the limit equilibrium methods.

Krahn (2003) also made the following remarks:

> "…the early developers of the method of slices recognised the limitations of computing realistic stresses on the slip surface. Lambe & Whitman (1969)…point out that the normal stress at a point acting on the slip surface should be mainly influenced by the weight of the soil lying above that point. This, they state, forms the basis of the method of slices. Morgenstern & Sangrey (1978) state that one of the uses '…of the factor of safety is to provide a measure of the average shear stress mobilized in the slope….This should not be confused with the actual stresses'. Unfortunately, these fundamental issues are sometimes forgotten as use of a method is generally adopted in routine practice."

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