HORIZONTAL SUBGRADE REACTION FOR CANTILEVERED RETAINING WALL ANALYSIS

GEO REPORT No. 21

W.K. Pun & P.L.R. Pang

GEOTECHNICAL ENGINEERING OFFICE
CIVIL ENGINEERING DEPARTMENT
THE GOVERNMENT OF THE HONG KONG
SPECIAL ADMINISTRATIVE REGION

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Civil Engineering Department,
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PREFACE

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A. W. Malone
Principal Government Geotechnical Engineer
April 1995

FOREWORD

As part of the R & D work to revise Geoguide 1: Guide to Retaining Wall Design, a review of the subject of horizontal subgrade reaction has been carried out.

The study was carried out by Mr W. K. Pun under the supervision of Dr P. L. R. Pang. This report documents the results of the study. Messrs. J. M. Shen and K. L. Siu provided useful comments on a draft version of the report.

(Y. C. CHAN)
Chief Geotechnical Engineer/Special Projects

CONTENTS

1. INTRODUCTION

As part of the R & D project to revise Geoguide 1: Guide to Retaining Wall Design (GCO, 1982), a review of the subject of horizontal subgrade reaction has been carried out. It has been found that while there is much guidance on soil deformation parameters to be used for laterally-loaded pile analysis, there is little guidance on similar parameters to be used for cantilevered sheet retaining wall analysis. For this reason, many designers in Hong Kong have been using a parameter known as the constant of horizontal subgrade reaction, n_h, introduced by Terzaghi (1955), for both pile and wall analyses. The approach of using n_h for wall analysis has been questioned in the past because while the behaviour of a pile is governed by its width (B), the behaviour of a wall is thought to be related to its depth of embedment (D).

In this report, values of soil deformation parameters relevant to wall analysis are derived using two different approaches, viz. the elastic solution approach and back-analysis of published results of passive earth pressure measurements. The results of the comparison exercises carried out to check the derived deformation parameter values are also presented.

2. LITERATURE REVIEW

The 'beam on elastic foundation' or 'subgrade reaction' approach is often used for analysing the internal forces and deformations of a cantilevered sheet retaining wall. In this approach, the wall is represented by a vertical elastic beam and the soil mass is modelled as a Winkler medium, for which displacement is proportional to pressure. Terzaghi (1955) characterised the 'stiffness' of the Winkler medium by a coefficient of horizontal subgrade reaction k_h:

$$k_h = \frac{p}{y} \qquad \text{. . . . . (1)}$$

where p = contact <u>pressure</u>
y = displacement

The S.I. units commonly used in the above equation are kN/m³, kPa and m (or MN/m³, MPa and m) for k_h, p and y respectively.

In actual design, it is convenient to consider the forces acting over a unit width of the wall and analyse the problem numerically. The Winkler medium is often discretised, i.e. represented by a series of linear elastic springs with spring stiffnesses or 'constants' k_s (Figure 1). The following equation may be used to evaluate k_s:

$$k_s = \frac{P}{y} = \frac{p\Delta z}{y} = k_h \Delta z \qquad \text{. . . . . . . (2)}$$

where P = reactive <u>force</u> (per unit width of the wall) in the spring at depth z below ground surface
y = lateral deflection of the wall at the spring location
Δz = length of wall over which the spring acts

The consistent S.I. units for k_s and P are in kPa (i.e. kN/m per m deflection) and kN/m respectively. The units of y, z and Δz are all in metres.

Rowe (1956a) proposed the following expression for k_h for sands having values of Young's modulus increasing linearly with depth:

$$k_h = m_h z/d \qquad \ldots\ldots\ldots\ldots \qquad (3)$$

where m_h = constant of horizontal subgrade reaction of the soil for wall analysis
z = depth below ground surface
d = depth of wall embedment in the soil

As shown in Figure 2, the parameter m_h is a secant modulus. Rowe (1956b) has recommended a range of m_h values for sands. These are reproduced in Table 1.

Terzaghi (1955) modelled the 'pressure-displacement' curve of a wall in a slightly different way. The curve is represented by two straight lines, which are associated with soil deformation parameters l_h and l_h' and an apparent earth pressure coefficient K_o'. The interpretation of l_h, l_h' and K_o' is given in Figure 2. The coefficient of horizontal subgrade reaction, k_h, is related to l_h and d, the depth of embedment of the wall in soil, by the following equation:

$$k_h = l_h z/d \qquad \ldots\ldots\ldots\ldots \qquad (4)$$

where l_h = constant of horizontal subgrade reaction for anchored bulkheads

Values of l_h, l_h' and K_o' recommended by Terzaghi (1955) for sands are reproduced in Table 2. While similar to m_h as defined by equation (3), it should be noted that a different datum, K_o', needs to be used with l_h.

3. DERIVATION OF THE CONSTANT OF HORIZONTAL SUBGRADE REACTION FOR WALL ANALYSIS

3.1 Derivation Based on Elastic Solution by Finn (1963)

Finn (1963) derived the pressure distribution acting on a smooth translating wall supporting a homogeneous, isotropic, elastic soil (Figure 3). The horizontal pressure P_t acting on the wall is given by the following equation:

$$P_t = \frac{\nu\gamma z}{1-\nu} + \frac{4zd}{(d+z)^3(d-z)} \cdot \frac{ydE_s}{(1-\nu^2)\pi} \qquad \ldots\ldots\ldots \qquad (5)$$

where E_s = Young's modulus of soil
d = total depth of wall
y = horizontal wall displacement towards the soil
z = depth measured from top of wall
γ = unit weight of soil
ν = Poisson's ratio

The first term in equation (5) is the at-rest earth pressure. The second term is the earth pressure p generated by a displacement y of the wall:

$$p = \frac{4zd}{(d+z)^3(d-z)} \cdot \frac{ydE_s}{(1-\nu^2)\pi} \qquad \ldots\ldots\ldots\ldots \qquad (6)$$

Rearranging the terms, the following is obtained:

$$\frac{pd}{yE_s} = \frac{4}{(1-\nu^2)\pi} \cdot \frac{z/d}{(1+z/d)^3(1-z/d)} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (7)$$

Equation (7) is plotted in Figure 4 for $\nu = 0.2$ and $\nu = 0.3$. It can be seen that the curves are insensitive to ν and for (z/d) up to 0.7, the curves are virtually linear. Equation (7) can thus be approximated by a straight line for the range of (z/d) from 0 to 0.7, with the following equation:

$$\frac{pd}{yE_s} = 0.87 \frac{z}{d} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (8)$$

or,

$$\frac{p}{y} = 0.87 \frac{E_s}{d^2} z \quad . \; . \; . \; . \; . \; . \; . \; . \quad (9)$$

By comparing equations (3) and (9) and noting that $k_h = p/y$, we have

$$m_h = 0.87 \frac{E_s}{d} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (10)$$

It should be noted that the m_h expression derived based on the homogeneous, isotropic, elastic soil model is not strain-dependent.

3.2 Derivation Based on Passive Earth Pressure Measurements by Rowe & Peaker (1965)

Rowe & Peaker (1965) reported the results of a series of model tests carried out to measure the passive earth pressure acting on a vertical wall translated against a mass of dry sand with a horizontal top surface. The wall in contact with the sand was 1.83 m (6 ft) wide and 0.46 m (1.5 ft) high. The wall was set to move against the sand at a given inclination to the horizontal without rotation. The passive earth pressure acting normal to the wall was measured by means of pressure cells.

Tests were conducted both for loose sand and dense sand. The loose state was achieved by directly pouring the sand from a rubber supply pipe, the outlet of which was maintained at a constant height above the sand level. The dense state was achieved by vibrating each 76 mm (3 inches) layer of sand with a pan vibrator. The properties of the sand are summarised in Table 3.

Table 4 tabulates the measured passive earth pressure coefficients, K_{pm}, mobilized at various values of wall displacements. The mobilized passive earth pressure coefficients can be related to m_h by the following equation (see Appendix A for derivation):

$$\frac{m_h}{K_p\gamma} = \frac{(K_{pm} - K_o)}{K_p} \Big/ (y/d) \quad . \; . \; . \quad (A8)$$

where K_o = coefficient of earth pressure at rest
K_p = coefficient of passive earth pressure
K_{pm} = mobilized passive earth pressure coefficient
d = depth of embedment of wall
m_h = constant of horizontal subgrade reaction for wall analysis
y = displacement of wall
γ = unit weight of soil

Based on Rowe & Peaker's measured earth pressure coefficients, values of

$(m_h/K_p\gamma)$ for various (y/d) ratio have been calculated using equation (11). The results are summarised in Table 5. In the calculation, K_o for loose sand and dense sand has been taken as 0.46 and 0.36 respectively. The largest measured value of K_{pm} has been taken as K_p.

Figures 5 and 6 show plots of $(m_h/K_p\gamma)$ versus (y/d) for loose sand and dense sand respectively. It can be seen that the data for a wide range of deformation conditions fall onto two unique curves. These curves take the shape of a hyperbola and can be fitted with the following equations:

$$\frac{m_h}{K_p\gamma} = \frac{0.64}{(y/d) + 0.017} \quad \text{for loose sand} \quad \ldots \quad (12)$$

$$\frac{m_h}{K_p\gamma} = \frac{1.09}{(y/d) + 0.011} \quad \text{for dense sand} \quad \ldots \quad (13)$$

In the above equations, the units of m_h and γ are the same.

As opposed to the homogeneous, isotropic, elastic soil model, the m_h expressions derived from Rowe & Peaker's results are strain-dependent.

4. COMPARISON WITH PUBLISHED WORK

4.1 Relevant Published Work

In order to check the validity of the m_h values derived in the previous Section, wall deformations predicted using the derived m_h values have been compared with measured behaviour. Since no suitable monitoring results of prototype walls are available, comparison has only been possible with the results of model tests. Two pieces of relevant experimental work could be found: one by James & Bransby (1970) and another by Bransby & Milligan (1975).

4.2 Comparison with Rotating Wall Experiment of James & Bransby (1970)

In the model tests carried out by James & Bransby (1970), a wall 190 mm (7.5 inches) wide and 307 mm (12.1 inches) high was rotated about its toe into a mass of dry sand with a horizontal top surface (Figure 7). Earth pressure cells were installed to measure the distribution of normal and shear stresses, as well as of the total forces and moments acting on the wall. The properties of the sand used in the tests are summarised in Table 6.

Two sets of test results are available: one on loose sand and another on dense sand. The test results were expressed by James & Bransby (1970) as dimensionless normal force coefficients P_c and depth coefficients D_c, which are defined below:

$$P_c = \frac{P}{\gamma w d^2} \quad \ldots \quad (14)$$

$$D_c = \frac{1}{d}\left(d - \frac{M}{P}\right) \approx \frac{D}{d} \quad \ldots \quad (15)$$

where D = depth of point of action of total force
M = bending moment acting on the wall about its toe
P = total normal force acting on the wall
d = initial wall height

w = width of the wall
γ = unit weight of the sand mass

James & Bransby's parameters P_c and D_c can be related to m_h as follows (see Appendix B for details):

$$P_c = 0.5K_o + 0.5(K_o - K_p)(\frac{h_c}{d})^2 + \frac{m_h\tan\theta}{\gamma}\left[\frac{1}{6} - \frac{1}{2}(\frac{h_c}{d})^2 + \frac{1}{3}(\frac{h_c}{d})^3\right] \qquad (16)$$

$$D_c = \frac{1}{P_c}\left\{\frac{1}{3}K_o + \frac{1}{3}(K_p - K_o)(\frac{h_c}{d})^3 + \frac{m_h\tan\theta}{\gamma}\left[\frac{1}{12} - \frac{1}{3}(\frac{h_c}{d})^3 + \frac{1}{4}(\frac{h_c}{d})^4\right]\right\} \qquad (17)$$

$$\frac{h_c}{d} = 1 - \frac{(K_p - K_o)\gamma}{m_h\tan\theta} \geq 0 \qquad . . . . (18)$$

where K_o = coefficient of earth pressure at rest
K_p = passive earth pressure coefficient
h_c = depth of sand mass over which full passive pressure is mobilized
θ = angle rotation of wall about its toe

In order to check the results of the previous Section, P_c and D_c values have been calculated using equations (16) to (18) with m_h values obtained from the empirical equations (12) and (13). Table 7 tabulates the empirical m_h values relevant to the model tests. In the calculation, the value of K_p has been evaluated using Caquot & Kerisel (1948)'s charts. The values of P_c and D_c predicted using the m_h values in Table 7 are given in Table 8. Prediction using m_h values based on the elastic solution has not been carried out because the Young's modulus of the sand is not known.

The calculated P_c and D_c values are compared to the measured values in Figure 8. This indicates that the prediction of P_c, which represents the total force acting on the wall, is very promising for both loose sand and dense sand for small movements (y/d up to about 5%). The prediction of D_c, which is related to the point of action of P_c, is not as good. This can be explained by examining the shape of the pressure distributions measured by James & Bransby. It appears that no pressure has been registered near the toe of the wall, resulting in measured D_c values somewhat higher than the predicted ones. This is probably due to arching around the hinge, which is not modelled by the subgrade reaction analysis.

4.3 Comparison with Sheet Pile Wall Experiment of Bransby & Milligan (1975)

The model tests carried out by Bransby & Milligan (1975) consisted of a number of sheet pile walls of different flexibility and roughness, some retaining dense sand and others retaining loose sand. The overall height of the wall was 300 mm. The width of the wall for tests in loose sand and in dense sand was 195 mm and 137 mm respectively. Figure 9 shows a schematic section of the model walls. Most of the tests were 'dredged' tests, i.e. the sand was poured to the full height simultaneously on both sides of the wall while the top of the wall was held fixed. Then the top of the wall was released and sand was removed gradually on one side of the wall until failure occurred. Displacements of the wall were recorded at different levels of excavation. The properties of the sand used in the tests are summarised in Table 9.

Figure 10 shows the measured and predicted deformations at various depths of excavation for two tests for which relatively stiff walls were used. The predicted deformations of the walls were obtained using a Winkler model with the empirical m_h expressions derived from Rowe & Peaker (1965)'s experimental results. The m_h values used in the analyses are summarised in Table 10. As neither wall friction nor passive pressure has been measured by Bransby & Milligan (1975), the same K_p values for the sand used in James & Bransby (1970)'s test have been taken for the purpose of evaluating m_h for the analyses. This assumption is considered reasonable because the sands used in the two experiments have very similar properties. Deformations at mid-depth of the embedded portion of the walls have been taken as the average displacement for the evaluation of m_h values to be used in the analysis. The soil-structure interaction analysis for the wall was carried out by means of a computer program for plane frame structural analysis known as "MICROFEAP".

It can be seen from Figure 10 that the prediction of deformations using the Winkler model with the derived m_h values is very promising. Prediction using m_h values based on the elastic solution has not been carried out because the Young's modulus of the sand is not known.

5. SELECTION OF DEFORMATION PARAMETERS FOR THE WINKLER MODEL FOR CANTILEVERED SHEET RETAINING WALL ANALYSIS

In Hong Kong, the deformation parameter n_h is commonly used for soil-structure interaction analysis of cantilevered sheet retaining walls. Reference is often made to the work of Terzaghi (1955), Reese et al (1974) and Elson (1984) for recommended values of n_h, which have been derived from data on single piles. For walls composed of piles (e.g. caissons) that are widely-spaced, the practice of carrying out analysis for each individual pile and to evaluate the spring constants using the parameter n_h is reasonable. However, the use of n_h for sheet walls or walls composed of closely-spaced piles is inappropriate. For example, for sheet wall analysis, the modelling of the subgrade reaction by a single series of springs for the whole wall and to evaluate the spring constants by n_h will give very conservative results. This is because the model assumes that the soil resistance for the whole wall is the same as that for a narrow pile. For this reason, this approach is not used in practice. Another approach is to assume the soil resistance (as calculated by the n_h values) acts over a unit width of the wall. This is arbitrary and will underestimate the deformation of the wall in most cases. From a consideration of the extent of the zone of influence of laterally-loaded piles, it is suggested that n_h should not be used for walls composed of piles spaced closer than about three times the pile diameter.

As indicated in Section 2, two soil deformation parameters, viz. l_h and m_h, have been proposed for use with the Winkler model for wall analysis. The pros and cons of using these parameters are given below.

Terzaghi's l_h values have been derived for flexible anchored bulkheads. They have to be used in conjunction with an earth pressure coefficient K_o', which is hard to determine in practice. Using the l_h and K_o' values suggested by Terzaghi (1955), soil-structure interaction analyses have been carried out to calculate the deformations of the stiff walls of Bransby & Milligan (1975). Again the computer program "MICROFEAP" was used. The calculated deformations are found to be larger than the measured values by two orders of magnitude. Hence, it appears that Terzaghi's recommended l_h and K_o' values for anchored bulkheads are not suitable for analysis of stiff cantilevered sheet retaining walls.

Three sets of m_h values are now available. The first set is that recommended by Rowe (1956b). Two additional expressions have been derived in the present study, one based on the elastic solution, and another based on the passive earth pressure measurements of Rowe & Peaker (1965).

As the parameter m_h is essentially a secant modulus, its values is strain dependent (Figure 2). The m_h values recommended by Rowe (1956b) appear to have been derived from a particular set of observations of very small deformations. The strain level for the dense sand case was less than 0.001 and that for the loose sand case was less than 0.005. These recommended m_h values are not appropriate for larger strain conditions. The m_h values derived in the present study, i.e. equations (10), (12) and (13), are more general. For the m_h expression based on the elastic solution, i.e. equation (10), a Young's modulus value corresponding to the appropriate strain level in the soil mass should be used.

In Sections 4.2 and 4.3, it has been shown that the use of the m_h expressions derived from Rowe & Peaker's experimental results can give reasonable predictions of the behaviour of model cantilevered sheet retaining walls. This provides a basis for proposing the use of the derived m_h expressions in actual design. The applicability of the derived m_h expressions should however be verified by monitoring the behaviour of prototype walls.

It should be noted that for realistic assessment of deformations, the K_p values to be used in conjunction with equations (12) and (13) should be best estimate values, rather than conservative design values. For example, for a dense sand with $\phi' = 45°$, $c' = 0$, $\delta = 0.75\phi'$ (K_p = 25 from Caquot & Kerisel's chart) and $\gamma = 20$ kN/m³, $m_h = 47$ MN/m³ for a y/d value of 0.001. For a loose sand with $\phi' = 30°$, $c' = 0$, $\delta = 0.75\phi'$ (K_p = 5.7 from Caquot & Kerisel's chart) and $\gamma = 18$ kN/m³, $m_h = 3$ MN/m³ for a y/d value of 0.005. These values are similar to the m_h values of 63 and 2.4 MN/m³ recommended by Rowe (1956b) for dense and loose sand respectively.

In the determination of internal forces and deformations for cantilevered sheet retaining walls, it is convenient to consider the forces acting over a unit width of the wall. The m_h expressions given by equations (12) and (13) can be used directly in subgrade reaction analyses. However, in the analysis of cantilevered retaining walls composed of closely-spaced piles (e.g. caissons), it is more convenient to consider the forces acting on the individual piles. Where the pile spacing (i.e. the distance between centres of adjacent piles) is not greater than about three times the pile diameter, the passive earth pressure may be assumed to act over the full pile spacing. For such cases, the m_h values should be multiplied by the pile spacing S instead of the pile width B in calculating the spring constants k_s.

The m_h expressions given above were derived based on data on sands. Also, the Winkler model is a crude approximation of actual soil behaviour. Nevertheless, given the uncertainties involved in the analysis, e.g. soil layering, homogeneity, shear strength, etc, it is often not meaningful to predict deformations other than their orders of magnitude. In this context, the use of the m_h expressions for soils derived from insitu rock weathering in Hong Kong should be acceptable, pending availability of well-documented case histories on actual performance.

6. CONCLUSIONS AND RECOMMENDATIONS

A review of the Winkler model, which is commonly used in Hong Kong for soil-structure interaction analysis of cantilevered sheet and caisson retaining walls has been carried out. Very often, the spring constants for the Winkler medium are evaluated using Terzaghi (1955)'s n_h values. However, these n_h values are derived from data on single piles and are inappropriate for wall analysis.

For sheet wall analysis, it is appropriate to consider the forces acting on a unit width of the wall. Two deformation parameters, viz. l_h by Terzaghi (1955) and m_h by Rowe (1956b), have been proposed for this type of analysis. Terzaghi's l_h values, which were derived for flexible anchored bulkheads, appear to be unsuitable for cantilevered sheet retaining wall analysis.

Two sets of expressions for m_h have been derived in the present study: a theoretical expression for m_h based on the elastic solution of Finn (1963) and empirical expressions based on the passive earth pressure measurements by Rowe & Peaker (1965). The latter expressions, which are strain-dependent, have been used to back-analyse the behaviour of model walls of James & Bransby (1970) and Bransby & Milligan (1975). The predicted behaviour have been found to compare well with the measurements. It is suggested that the empirical expressions for m_h given by equations (12) and (13) can be used for soil-structure interaction analysis of cantilevered sheet and caisson retaining walls. The m_h values used in design should be verified by monitoring the actual wall performance.

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LIST OF TABLES

Table 1 - Values of m_h Recommended by Rowe (1956b)

Relative Density	Loose	Medium Dense	Dense
Approximate Unit Weight (kN/m^3)	14.5	-	17.3
m_h (flexible walls)	8.8	31.4	125
m_h (stiff walls)	2.4	-	63
Note : Unit of m_h is MN/m^3.			

Table 2 - Values of l_h, l_h' and K_o' Recommended by Terzaghi (1955)

Relative Density	Loose	Medium Dense	Dense
SPT 'N' Value	4 - 10	10 - 30	30 - 50
l_h (dry sand)	0.8	2.5	6.3
l_h (submerged sand)	0.5	1.6	4.1
l_h' (dry sand)	39.3	62.9	94.3
l_h' (submerged sand)	23.6	37.7	56.6
K_o'	0.4	0.8	1.2
Note : Units of l_h and l_h' are MN/m^3.			

Table 3 - Properties of Sand Used in the Model Tests of Rowe & Peaker (1965)

State	Loose	Dense
Specific gravity	2.655	2.655
Porosity	0.42	0.37
Unit weight (kN/m^3)	15.1	16.4
Maximum void ratio	0.786	0.786
Minimum void ratio	0.539	0.539
Insitu void ratio	0.70	0.59
Relative density	25%	80%
Friction angle (triaxial test)	33°	39.5°

Table 4 - Mobilized Passive Earth Pressure Coefficients Derived from Data Given by Rowe & Peaker (1965)

State	Loose					Dense					
θ (degrees)	-45	0	10	20	45	-45	-25	0	10	20	65
y/d	K_{pm}										
0	0.46	0.46	0.46	0.46	0.46	0.36	0.36	0.36	0.36	0.36	0.36
0.01	1.0	1.4	1.0	1.9	1.9	2.3	3.3	3.5	4.2	4.3	4.2
0.02	1.1	1.9	1.6	2.5	3.2	3.1	4.3	4.5	5.4	5.4	5.3
0.03	1.2	2.2	1.9	2.8	3.9	2.9	4.5	5.6	6.0	6.1	6.1
0.04	1.3	2.4	2.2	3.1	4.3	2.5	4.6	6.0	6.5	6.6	6.6
0.05	1.3	2.6	2.4	3.2	4.8	2.1	4.6	6.0	6.8	7.0	7.2
0.075	1.4	3.1	2.8	3.7	5.2	1.7	3.7	5.9	6.8	7.4	8.1
0.10	1.4	3.4	3.2	4.1	5.6	1.3	3.3	5.9	6.7	7.3	8.6
0.15	1.4	3.8	3.7	4.7	6.1	1.1	-	-	6.5	7.1	6.4
0.20	-	4.1	4.2	5.3	6.5	-	-	-	-	6.7	-
0.25	-	4.3	4.6	5.7	6.6	-	-	-	-	-	-
0.30	-	-	5.0	6.1	-	-	-	-	-	-	-
0.35	-	-	5.2	6.4	-	-	-	-	-	-	-

Legend :

K_{pm} Mobilized passive earth pressure coefficient
d Height of wall
y Horizontal displacement of wall
θ Pre-set inclination of wall movement

Table 5 - Values of $m_h/K_p\gamma$ Calculated Using Mobilized Passive Earth Pressure Coefficients Given in Table 4

State	Loose					Dense					
θ(degrees)	-45	0	10	20	45	-45	-25	0	10	20	65
K_p	1.4	4.3	5.2	6.4	6.6	3.1	4.6	6.0	6.8	7.4	8.6
y/d	$m_h/K_p\gamma$										
0.01	38.5	21.8	10.3	22.4	21.7	72.4	64.1	35.7	56.6	53.4	44.7
0.02	22.8	16.7	10.9	15.9	20.7	44.3	42.9	34.6	37.1	34.1	28.8
0.03	17.5	13.4	9.2	12.1	17.3	27.3	30.0	29.2	27.7	25.9	22.3
0.04	14.0	11.2	8.3	10.2	14.5	17.3	23.1	23.5	22.6	21.1	18.1
0.05	11.9	9.9	7.4	8.5	13.1	11.2	18.4	18.8	19.0	18.0	15.9
0.075	8.4	8.1	5.9	6.7	5.9	5.7	9.7	12.3	12.6	12.7	12.0
0.10	6.7	6.8	5.2	5.6	7.7	3.0	6.4	9.2	9.3	9.4	9.4
0.15	4.4	5.1	4.1	4.4	5.6	1.5	-	-	6.0	6.0	4.6
0.20	-	4.2	3.5	3.7	4.5	-	-	-	-	4.2	-
0.25	-	3.5	3.1	3.2	3.7	-	-	-	-	-	-
0.30	-	-	2.9	2.9	-	-	-	-	-	-	-
0.35	-	-	2.6	2.6	-	-	-	-	-	-	-

Notes : (1) See Table 4 for legend
(2) The largest measured value of Kpm has been taken as K_p.
(3) K_o for loose and dense sand has been taken as 0.46 and 0.36 respectively.

Table 6 - Properties of Sand Used in the Model Tests of James & Bransby (1970)

State	Loose	Dense
Specific gravity	2.66	2.66
Initial void ratio	0.769	0.505
Unit weight (kN/m^3)	14.7	17.3
Maximum void ratio	0.79	0.79
Minimum void ratio	0.49	0.49
Relative density	7%	95%
Friction angle	35°	49°
Wall friction angle	25°	37°
K_o	0.43	0.25

Table 7 - m_h Values Relevant to the Model Tests of James & Bransby (1970)

θ (degree)	y/d	m_h (MN/m³)	
		Loose	Dense
1	0.009	3.2	45
2	0.017	2.4	32
3	0.026	1.9	24
4	0.035	1.6	19
5	0.044	1.4	16
6	0.053	1.2	
7	0.061	1.1	
8	0.070	0.95	
9	0.079	0.86	
10	0.088	0.79	
11	0.097	0.72	
12	0.106	0.67	

Notes : (1) m_h values have been calculated using equations (12) and (13), with K_p = 8.8, γ = 14.7 kN/m³ for loose sand and K_p = 47, γ = 17.3 kN/m³ for dense sand.

(2) K_p values have been obtained using Caquot & Kerisel's charts.

(3) For dense sand, the empirical m_h values are only reliable up to a y/d value of about 0.05. Hence values for larger y/d have not been calculated.

Table 8 - Predicted P_c and D_c Values for the Model Tests of James & Bransby (1970)

θ (degrees)	Loose Sand			Dense Sand		
	h_c/d	P_c	D_c	h_c/d	P_c	D_c
1	0	0.83	0.54	0	7.71	0.50
2	0	1.18	0.53	0.28	10.73	0.51
3	0	1.36	0.53	0.36	11.78	0.52
4	0	1.49	0.52	0.40	12.31	0.52
5	0	1.57	0.52	0.43	12.73	0.53
6	0	1.62	0.52			
7	0.05	1.68	0.52			
8	0.09	1.75	0.52			
9	0.11	1.77	0.52			
10	0.13	1.82	0.52			
11	0.15	1.85	0.52			
12	0.15	1.85	0.52			

Notes : (1) h_c/d, P_c and D_c values have been calculated using equations (16) to (18), with m_h values taken from Table 7.
(2) For loose sand, $K_o = 0.43$, $K_p = 8.8$, $\gamma = 14.7$ kN/m³ have been used.
(3) For dense sand, $K_o = 0.25$, $K_p = 47$, $\gamma = 17.3$ kN/m³ have been used.
(4) For dense sand, P_c and D_c have been calculated only up to $\theta = 5°$ because the empirical m_h values for dense sand are unreliable for larger deformations.

Table 9 - Properties of Sand Used in the Model Tests of Bransby & Milligan (1975)

State	Loose	Dense
Specific gravity	2.65	2.65
Initial void ratio	0.78	0.55
Unit weight (kN/m³)	14.6	16.8
Maximum void ratio	0.79	0.79
Minimum void ratio	0.49	0.49
Relative density	3%	80%
Friction angle	35°	49°
K_a	0.27	0.14
K_o	0.43	0.25

Table 10 - m_h Values Relevant to the Model Tests of Bransby & Milligan (1975)

H_e/H	Measured y/d		m_h (MN/m³)	
	Loose sand	Dense sand	Loose sand	Dense sand
0.40	0.0005	-	4.7	-
0.45	0.011	-	2.9	-
0.50	0.024	0.0004	2.0	49
0.55	0.037	-	1.5	-
0.60	0.077	0.0018	0.9	46
0.65	-	-	-	-
0.70	-	0.0035	-	42

Legend :

H_e Depth of excavation in front of wall
H Overall Height of wall
d Depth of embedment
y Deformation at mid-depth of embedded portion of wall

Notes : (1) m_h values have been calculated using equations (12) and (13), with $K_p = 8.8$, $\gamma = 14.6$ kN/m³ for loose sand and $K_p = 47$, $\gamma = 16.8$ kN/m³ for dense sand.

(2) Neither wall friction nor passive earth pressure has been measured by Bransby & Milligan (1975). The values of K_p used are for a rough wall based on the test results of James & Bransby (1970).

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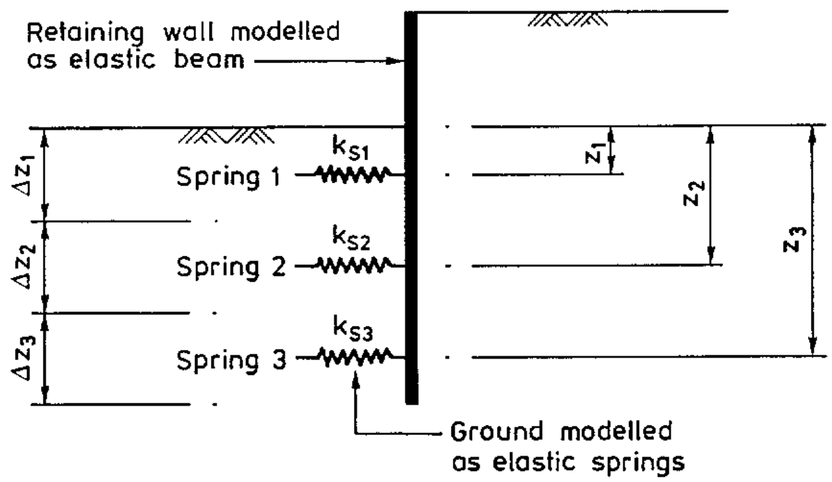
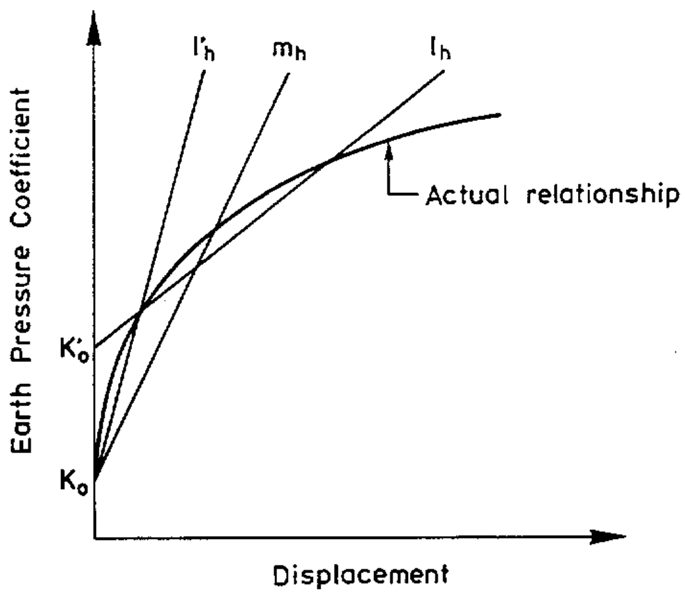


Legend :

k_{si}	Spring constants
z_i	Depth of spring below ground surface
Δz_i	Length of beam served by spring

Figure 1 - Winkler's Model for Cantilevered Sheet Retaining Wall Analysis



Legend :

K_o	Earth pressure coefficient at rest
K_o'	Apparent earth pressure coefficient at zero displacement
l_h, l_h'	Constants of horizontal subgrade reaction for anchored bulkhead
m_h	Constant of horizontal subgrade reaction for sheet walls

Notes : (1) Parameters K_o, K_o', l_h and l_h' are based on Terzaghi (1955).
(2) Parameter m_h is based on Rowe (1956a).

Figure 2 - Interpretation of Parameters l_h, l_h' and m_h

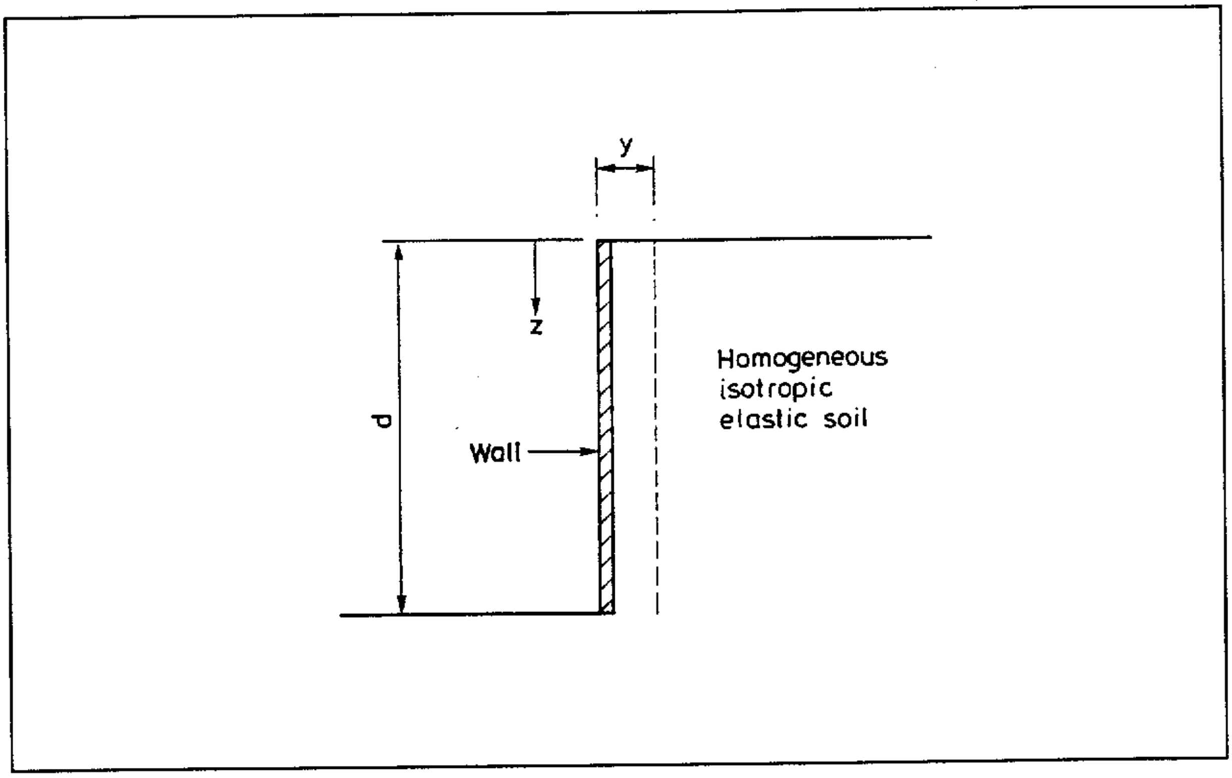
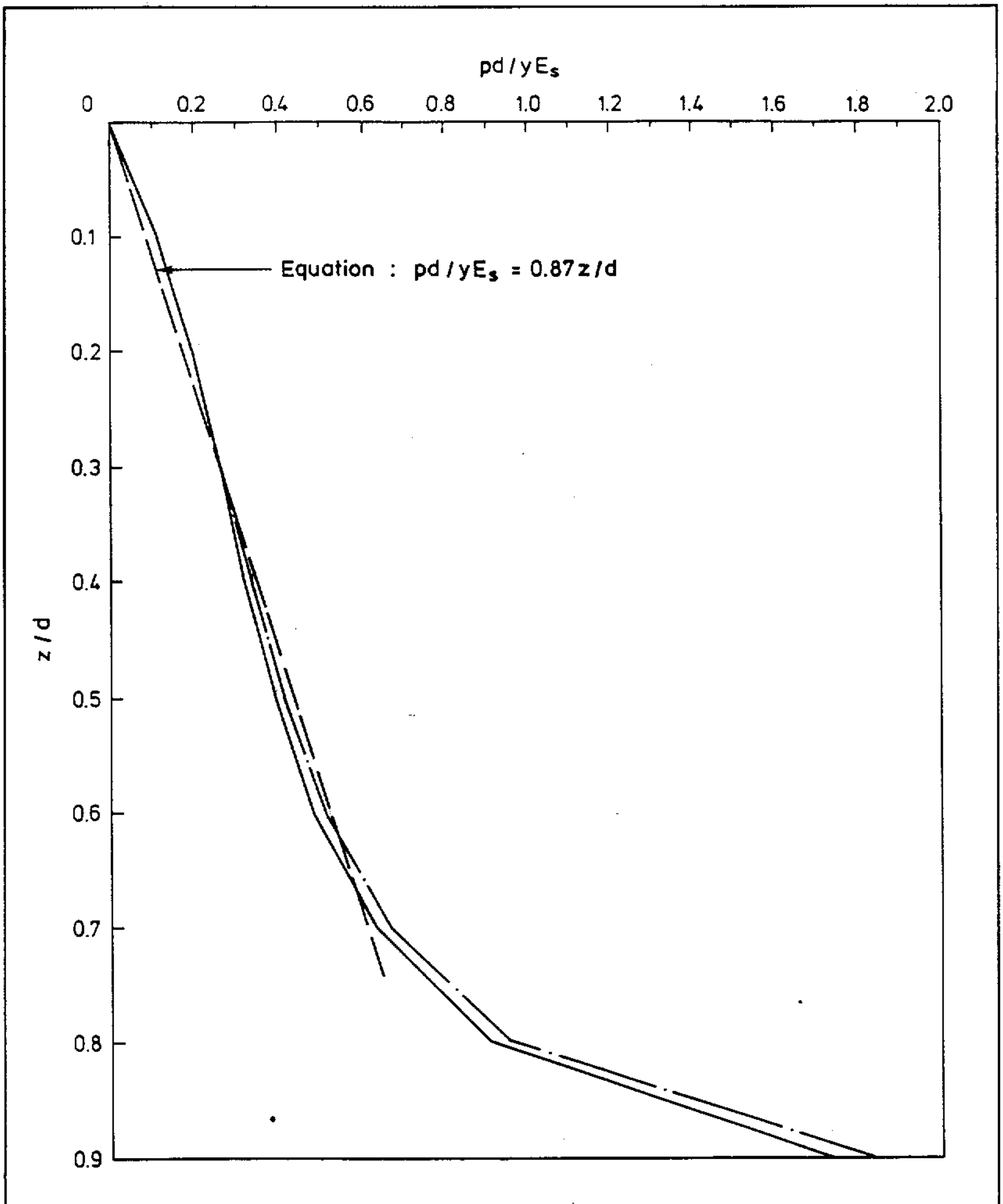


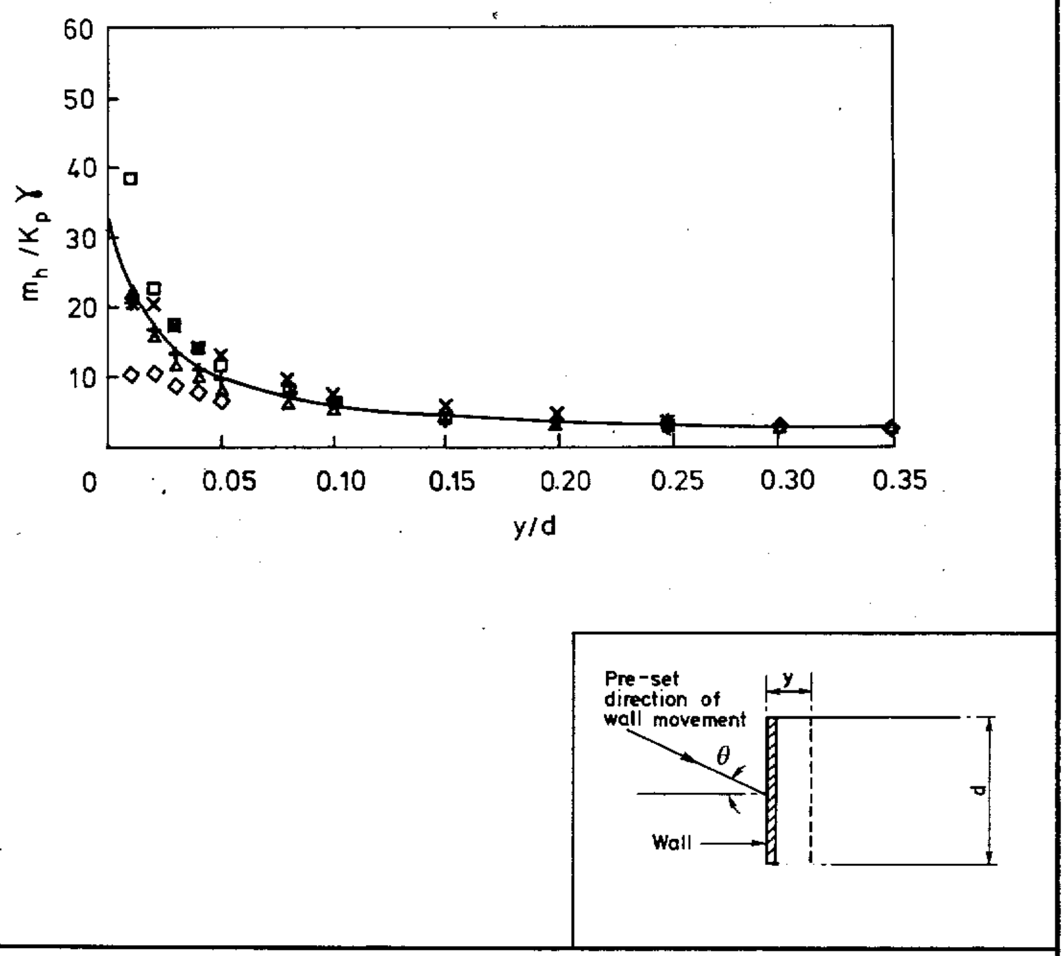
Figure 3 - Translating Wall Problem Considered by Finn (1963)



Legend :

E_s	Young's modulus of soil	d	Total depth of wall
ν	Poisson's ratio	p	Soil pressure
——	$\nu = 0.2$	y	Horizontal wall displacement
—·—	$\nu = 0.3$	z	Depth measure from top of wall

Figure 4 - Pressure Distribution on a Rigid Translating Wall Based on the Elastic Solution by Finn (1963)



Legend :

K_p	Coefficient of passive earth pressure	+	θ = 0°
d	Depth of embedment of wall	×	θ = 45°
m_h	Constant of horizontal subgrade reaction for wall analysis	▲	θ = 20°
y	Displacement of wall	◘	θ = -45°
γ	Unit weight of soil	◇	θ = 10°
θ	Pre-set angle of movement of wall to the horizontal (downward positive)		

Figure 5 - m_h Values for Loose Sand Derived from Rowe & Peaker (1965)'s Experimental Results

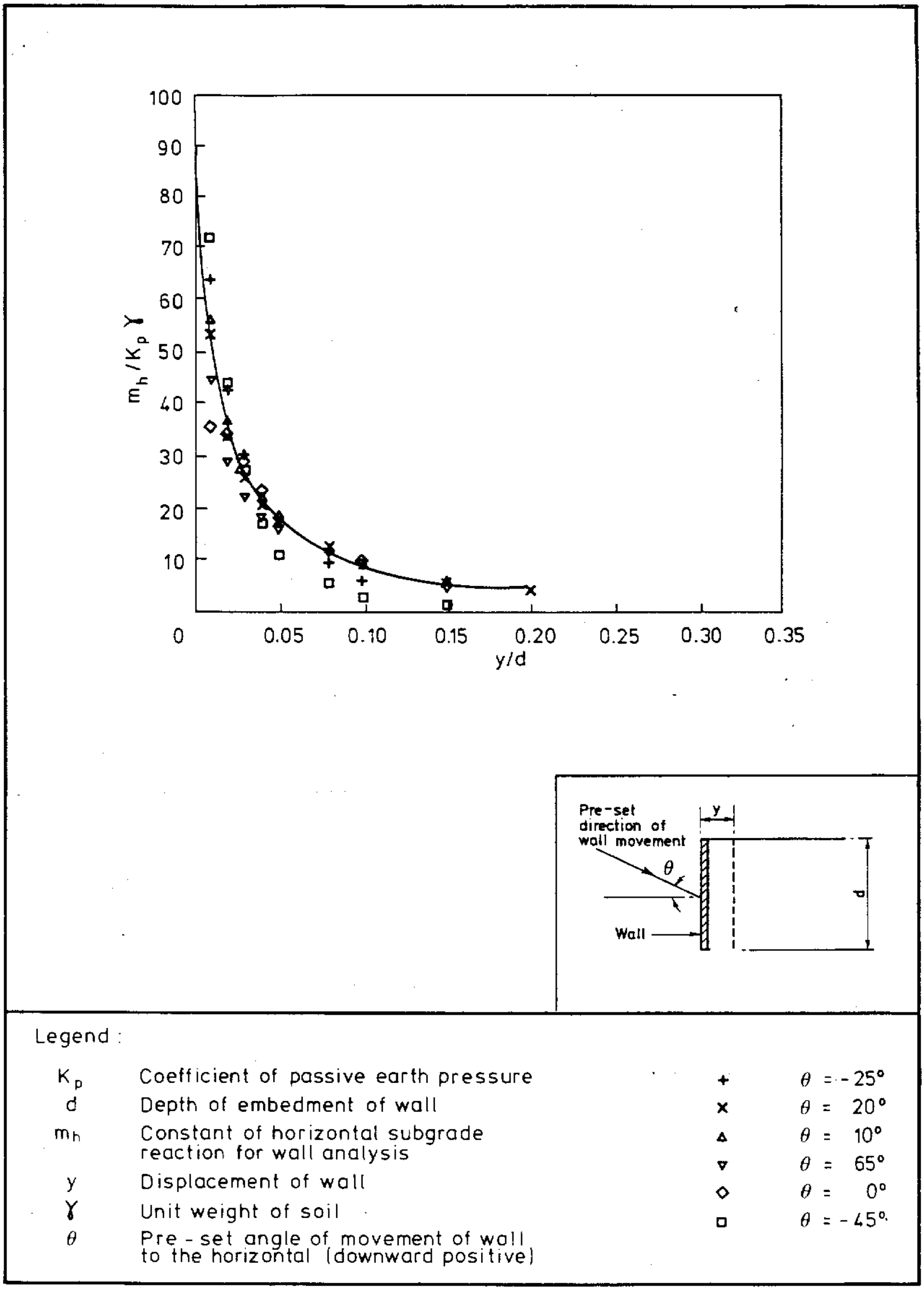


Figure 6 – m_h Values for Dense Sand Derived from Rowe & Peaker (1965)'s Experimental Results

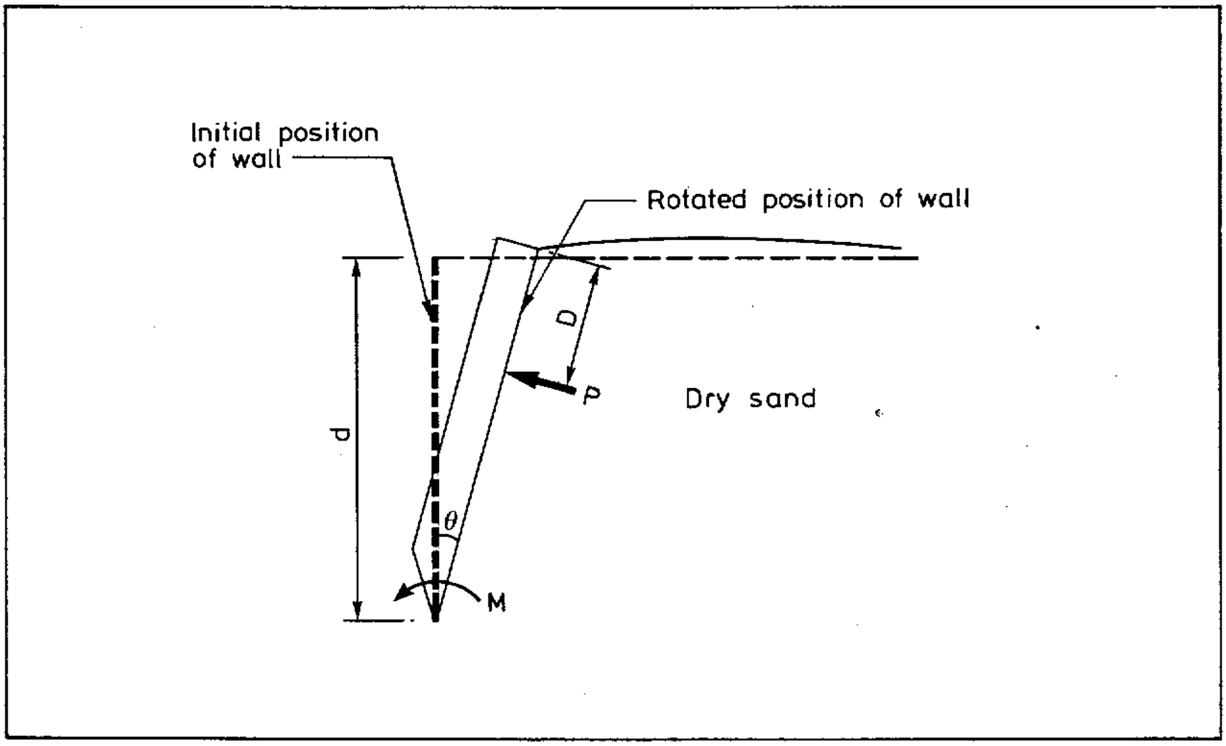


Figure 7 - Schematic Section of Model Wall in the Experiment of James & Bransby (1970)

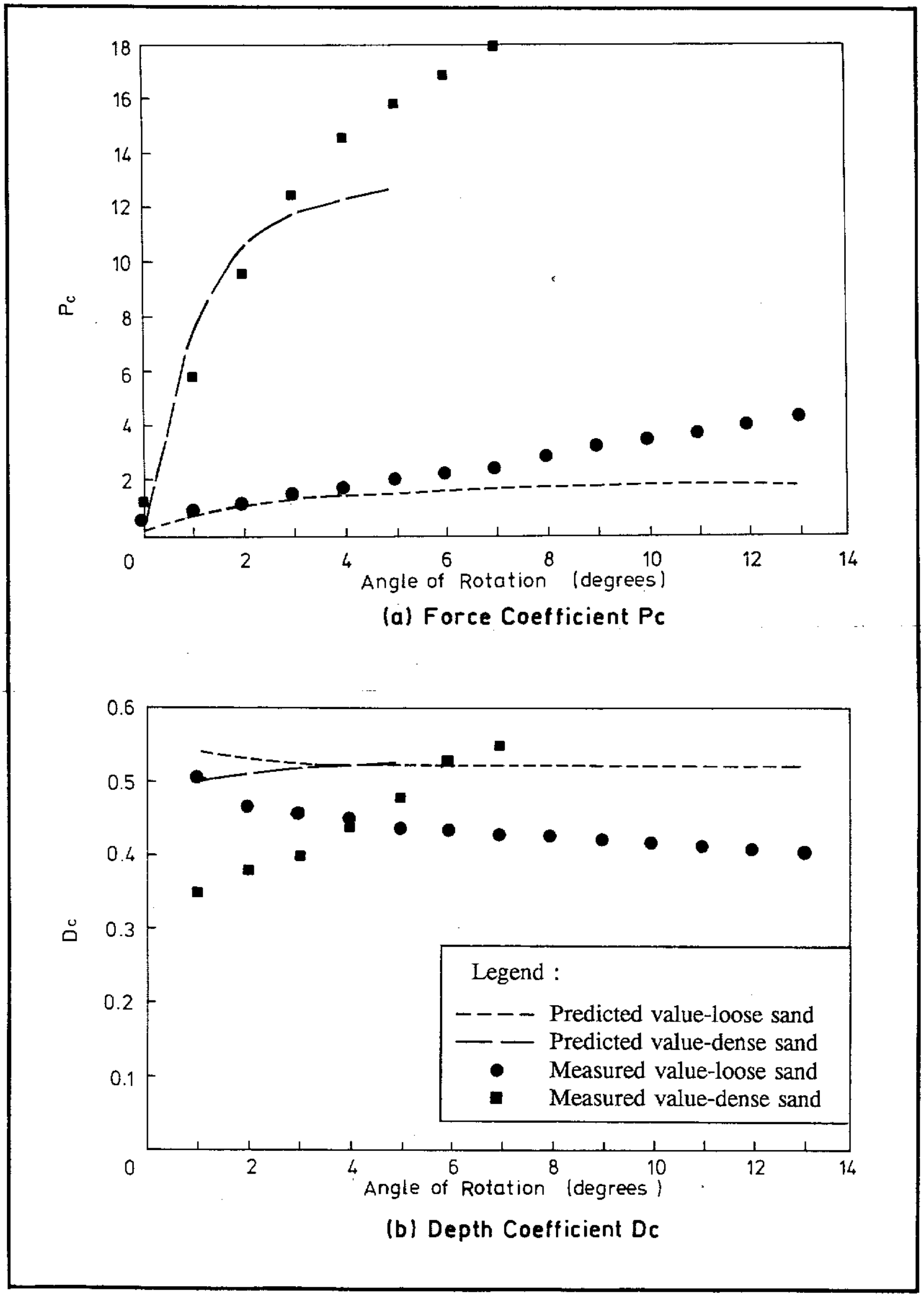


Figure 8 - Comparison of Predicted and Measured P_c and D_c Values for the Model Test of James & Bransby (1970)

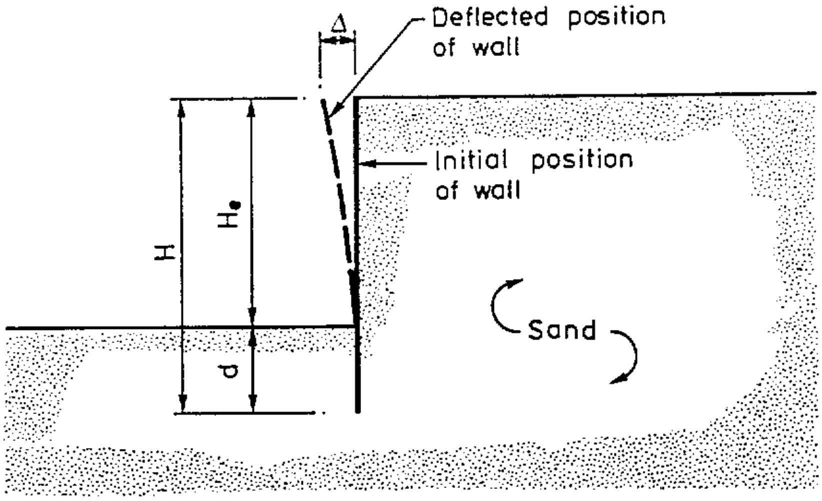
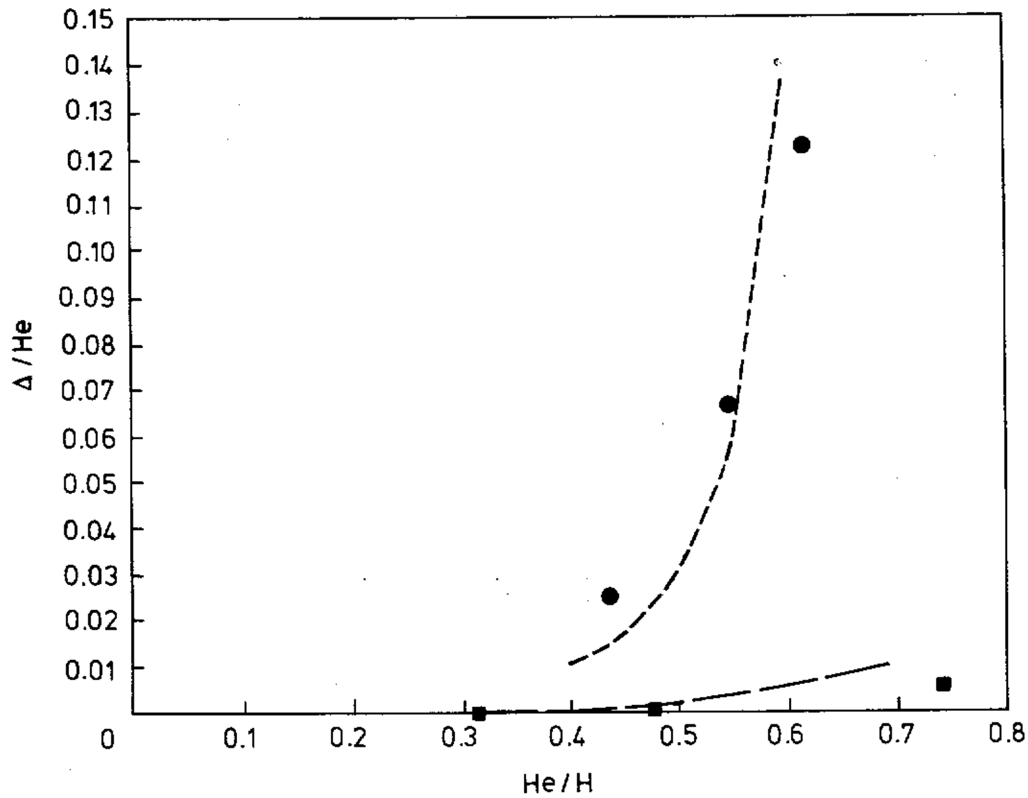


Figure 9 - Schematic Section of Model Wall in the Experiment of Bransby & Milligan (1975)



Legend :

----	Predicted deformation-loose sand	Δ	Deflection at the top of the wall
— —	Predicted deformation-dense sand	He	Depth of excavation
●	Measured deformation-loose sand	H	Overall height of the wall
■	Measured deformation-dense sand		

Figure 10 - Comparison of Predicted and Measured Wall Deformations for the Model Tests of Bransby & Milligan (1975)

APPENDIX A

DERIVATION OF THE RELATIONSHIP BETWEEN THE CONSTANT OF HORIZONTAL SUBGRADE REACTION AND DISPLACEMENT FOR A RIGID TRANSLATING WALL

A. DERIVATION OF THE RELATIONSHIP BETWEEN THE CONSTANT OF HORIZONTAL SUBGRADE REACTION AND DISPLACEMENT FOR A RIGID TRANSLATING WALL

Consider unit width of a rigid wall translated horizontally into a soil mass with a uniform displacement y as shown in Figure A1 (a). The total force P_w acting on the wall due to the earth pressure shown in Figure A1(b) is given by:

$$P_w = 0.5K_{pm}\gamma d^2 \qquad \text{(A1)}$$

where K_{pm} = mobilized passive earth pressure coefficient
d = height of wall
γ = unit weight of soil

For the Winkler model shown in Figure A1(c), the spring constants k_{si} for the springs are given by:

$$k_{si} = \frac{m_h z \Delta z}{d} \qquad \text{(A2)}$$

The total force P_s in the springs is given by:

$$P_s = \Sigma\, k_{si} y$$

$$= \Sigma\, \frac{m_h z \Delta z y}{d}$$

$$= y\int_0^d \frac{m_h z}{d}\, dz$$

$$= 0.5 m_h y d \qquad \text{(A4)}$$

Before movement occurs (i.e. at y = 0), there is an initial force P_o given by:

$$P_o = 0.5K_o\gamma d^2 \qquad \text{(A5)}$$

where K_o = coefficient of earth pressure at rest

Hence total force P_w acting on the wall is

$$P_w = 0.5m_h y d + 0.5K_o\gamma d^2 \qquad \text{(A6)}$$

Equating equations (A1) and (A6), we have:

$$0.5m_h y d + 0.5K_o\gamma d^2 = 0.5K_{pm}\gamma d^2$$

giving

$$m_h = \frac{(K_{pm} - K_o)\gamma d}{y} \qquad \text{(A7)}$$

Transposing the terms, the following non-dimensional relationship is obtained:

$$\frac{m_h}{K_p\gamma} = \frac{(K_{pm} - K_o)}{K_p} \Big/ (y/d) \qquad \text{(A8)}$$

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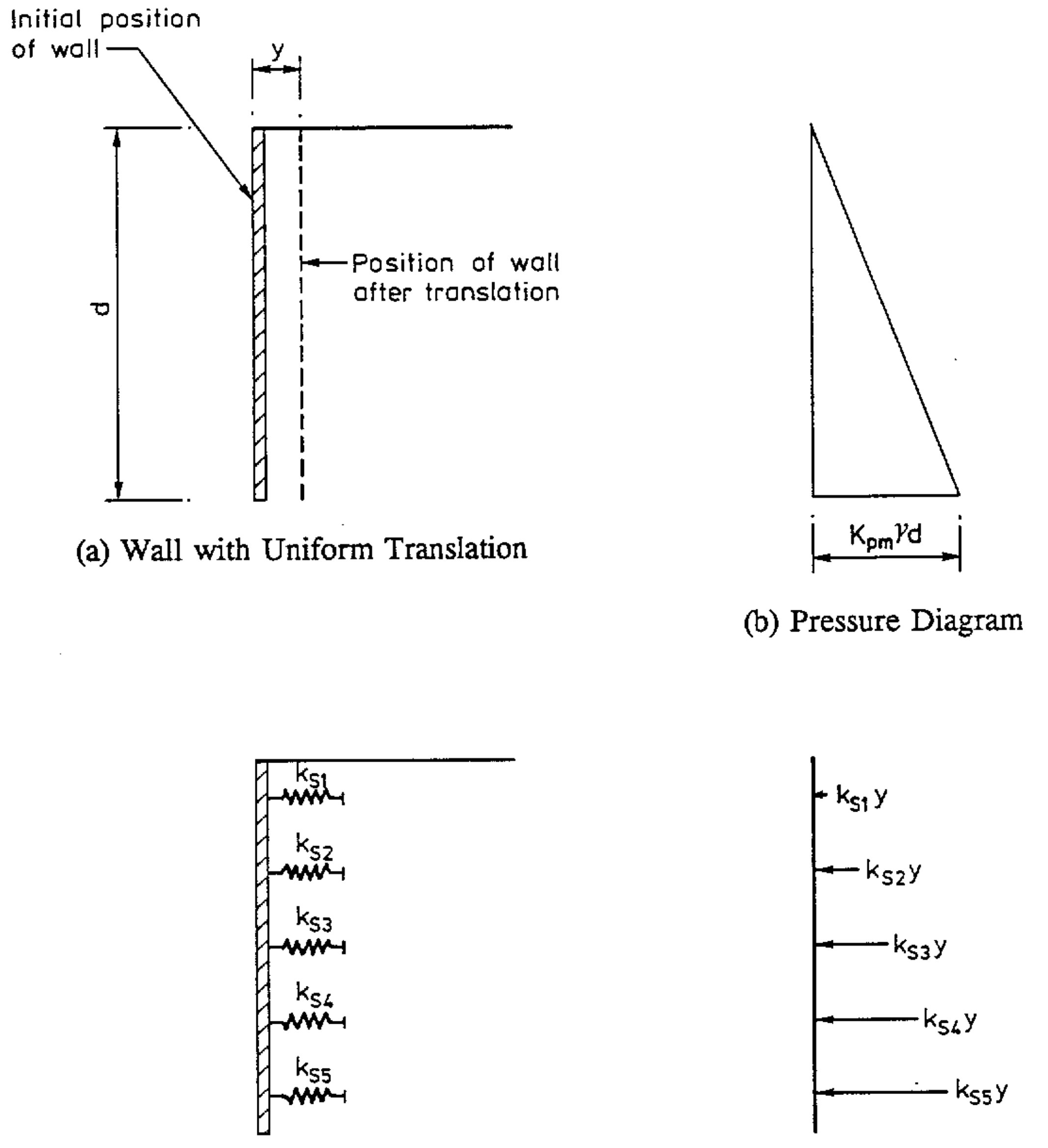

(a) Wall with Uniform Translation

(b) Pressure Diagram

(c) Spring Model

(d) Forces in Springs

Legend :

K_{pm}	Mobilized passive earth pressure coefficient
k_{si}	Spring constants
γ	Unit weight of soil

Figure A1 - Spring Model and Forces Acting on a Wall Translating into a Soil Mass

APPENDIX B

DERIVATION OF THE RELATIONSHIP BETWEEN JAMES & BRANSBY (1970)'S COEFFICIENTS AND THE CONSTANT OF HORIZONTAL SUBGRADE REACTION

B. DERIVATION OF THE RELATIONSHIP BETWEEN JAMES & BRANSBY (1970)'S COEFFICIENTS AND THE CONSTANT OF HORIZONTAL SUBGRADE REACTION

Consider a rigid wall rotating about its toe as shown in Figure B1 (a). For an angle of rotation θ, the displacement at the top of the wall is Δ. The displacement decreases linearly down the wall. At any depth z measured from the top of the wall, the displacement δ is given by:

$$\delta = (1 - \frac{z}{d})\Delta \quad \ldots\ldots\ldots \quad (B1)$$

The spring model for the wall is shown in Figure B1(b), and the pressure distribution is shown in Figure B1(c). At any depth z, the at-rest pressure p_o is given by:

$$p_o = K_o \gamma z \quad \ldots\ldots\ldots \quad (B2)$$

where K_o = coefficient of earth pressure at rest
γ = unit weight of soil

The spring pressure p_s is given by:

$$p_s = \frac{m_h z \delta}{d} = \frac{m_h z}{d}(1 - \frac{z}{d})\Delta \quad \ldots\ldots\ldots \quad (B3)$$

where m_h = constant of horizontal subgrade reaction

The total pressure p is therefore given by:

$$p = p_o + p_s \quad \ldots\ldots\ldots \quad (B4)$$

p can have a maximum value equal to the passive earth pressure p_p:

$$p_p = K_p \gamma z \quad \ldots\ldots\ldots \quad (B5)$$

where K_p = coefficient of passive earth pressure

The full passive earth pressure is mobilized down to a depth h_c, which is given by equating equations (B4) and (B5):

$$K_o \gamma h_c + \frac{m_h h_c}{d}(1 - \frac{h_c}{d})\Delta = K_p \gamma h_c$$

giving

$$\frac{h_c}{d} = 1 - \frac{(K_p - K_o)\gamma}{m_h \tan\theta} \quad \ldots \quad (B6)$$

The total force P_w acting on the wall is the sum of that due to full passive pressure down to depth h_c and that due to the spring forces and at-rest pressure at depth below h_c.

$$P_w = 0.5K_p \gamma w h_c^2 + 0.5K_o \gamma w(d^2 - h_c^2) + w\int_{h_c}^{d} \frac{m_h z \delta}{d}\,dz$$

$$= 0.5K_o \gamma w d^2 + 0.5(K_p - K_o)\gamma w h_c^2 + m_h w \tan\theta(\frac{1}{6}d^2 - \frac{1}{2}h_c^2 + \frac{1}{3}\frac{h_c^3}{d}) \quad (B7)$$

where w = width of wall

James & Bransby's normal force coefficient P_c is defined as $P_w/\gamma wd^2$. From equation (B7), Pc is given by:

$$P_c = 0.5K_o + 0.5(K_o - K_p)(\frac{h_c}{d})^2 + \frac{m_h \tan\theta}{\gamma}\left[\frac{1}{6} - \frac{1}{2}(\frac{h_c}{d})^2 + \frac{1}{3}(\frac{h_c}{d})^3\right] \qquad \text{(B8)}$$

The total moment M about the top of the wall is given by:

$$M = 0.5K_o\gamma wd^2\ \frac{2d}{3} + 0.5(K_p - K_o)\gamma wh_c^2\ \frac{2h_c}{3} + w\int_{h_c}^{d} \frac{m_h z^2 \delta}{d}\, dz$$

$$= \frac{1}{3}K_o\gamma wd^3 + \frac{1}{3}(K_p - K_o)\gamma wh_c^3 + m_h w\tan\theta(\frac{1}{12}d^3 - \frac{1}{3}h_c^3 + \frac{1}{4}\frac{h_c^4}{d}) \qquad \text{(B9)}$$

The point of action of the normal force on the wall acts at a depth D given by:

$$D = M/P_w \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \qquad \text{(B10)}$$

James & Bransby's depth coefficient D_c is approximately D/d. From equation (B10), D_c is given by:

$$D_c = \frac{1}{P_w d}\left[\frac{1}{3}K_o\gamma wd^3 + \frac{1}{3}(K_p - K_o)\gamma wh_c^3 + m_h w\tan\theta(\frac{1}{12}d^3 - \frac{1}{3}h_c^3 + \frac{1}{4}\frac{h_c^4}{d})\right]$$

$$= \frac{1}{P_c}\left\{\frac{1}{3}K_o + \frac{1}{3}(K_p - K_o)(\frac{h_c}{d})^3 + \frac{m_h \tan\theta}{\gamma}\left[\frac{1}{12} - \frac{1}{3}(\frac{h_c}{d})^3 + \frac{1}{4}(\frac{h_c}{d})^4\right]\right\} \qquad \text{(B11)}$$

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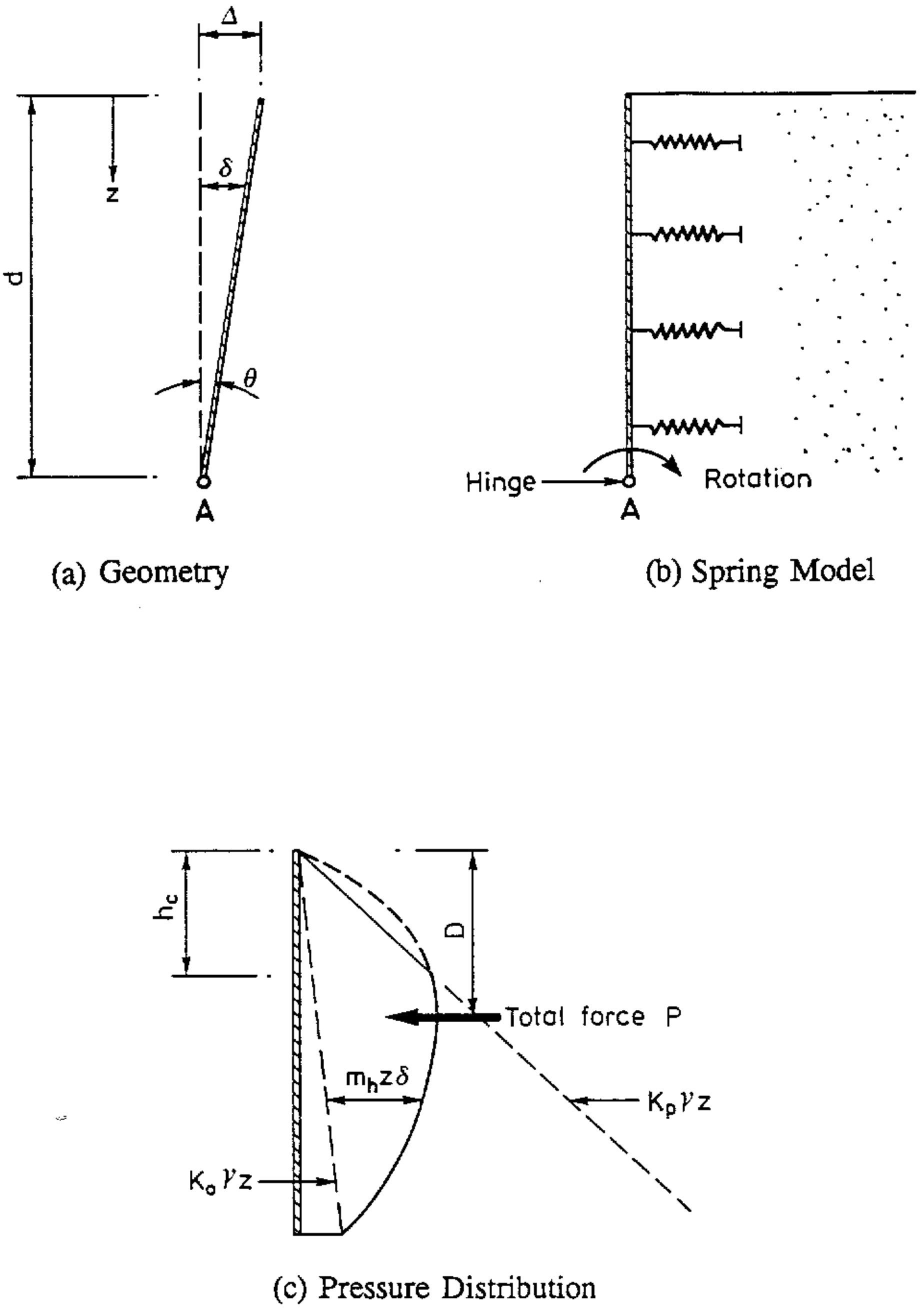

Legend :

K_o	At-rest earth pressure coefficient
K_p	Passive earth pressure coefficient
m_h	Constant of horizontal subgrade reaction for wall analysis

Figure B1 - Spring Model and Pressure Distribution for the Model Tests of James & Bransby (1970)