APPENDIX B

WORKED EXAMPLES

CONTENTS

B.1 FILL QUANTITY

Reference Section 5.2.

Given

Significant wave height H_s = 2.0 m

D_{50} of native sand = 0.3 mm

D_{50} of recharged sand = 0.5 mm

Mean sea level MSL = +1.3 mPD

Crest level = +3.0 mPD

Required beach width at mean sea level Y = 30.0 m

Find

Estimate the fill quantity.

Solution

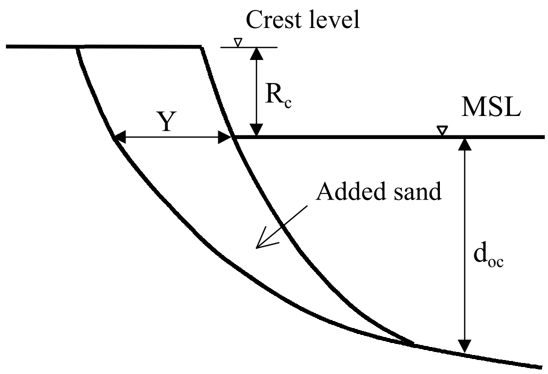
(1) Equilibrium Profile Method

$R_c = 3.0 - 1.3 = 1.7$ m

$d_{oc} = 1.75\, H_s = 1.75\,(2.0) = 3.5$ m

$A_N = 0.21\, D^{0.48} = 0.21\,(0.3)^{0.48} = 0.1178$

$A_R = 0.21\, D^{0.48} = 0.21\,(0.5)^{0.48} = 0.1506$



(Note : Subscripts N and R denote native sand and recharge sand respectively.)

$$Y\left(\frac{A_N}{d_{oc}}\right)^{3/2} + \left(\frac{A_N}{A_R}\right)^{3/2}$$

$$= (30.0)\left(\frac{0.1178}{3.5}\right)^{3/2} + \left(\frac{0.1178}{0.1506}\right)^{3/2}$$

$$= 0.877 < 1$$

Therefore, the profile is intersecting.

The volume of fill required

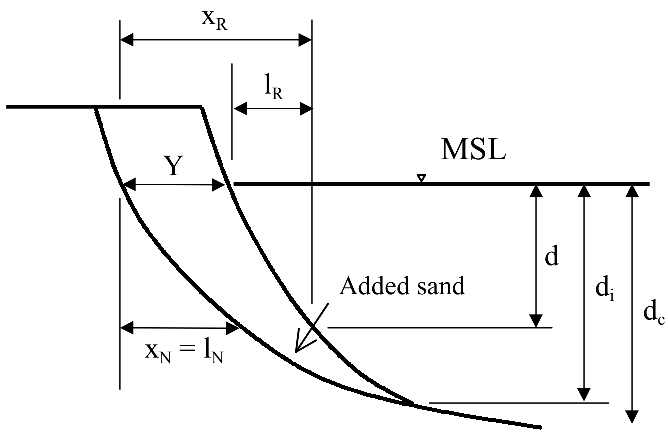
$$= \; R_c Y + \frac{A_N Y^{5/3}}{\left[1-\left(\dfrac{A_N}{A_R}\right)^{3/2}\right]^{2/3}}$$

$$= \; (1.7)(30.0) + \frac{(0.1178)(30.0)^{5/3}}{\left[1-\left(\dfrac{0.1178}{0.1506}\right)^{3/2}\right]^{2/3}}$$

$= 125.8\ m^3/m$

(2) Equilibrium Slope Method

The recharged profile is governed by the following equation :



$$w_N = 14D^{1.1} = 14\,(0.3)^{1.1} = 3.724\ \text{cm/s}$$

$$w_R = 14D^{1.1} = 14\,(0.5)^{1.1} = 6.531\ \text{cm/s}$$

$$l_R = \left(\frac{w_N}{w_R}\right)^{0.56} l_N = \left(\frac{3.724}{6.531}\right)^{0.56} l_N = 0.7301\, l_N$$

The existing beach profile is obtained from sounding survey results, or may be approximated by the following equation :

$$d = A_N x_N^{\ 2/3}$$

$$= 0.21\, D^{\,0.48}\, x_N^{\ 2/3}$$

$$= 0.21\,(0.3)^{0.48}\, x_N^{\ 2/3}$$

$$= 0.1178\, x_N^{\ 2/3}$$

or $\quad x_N = \left(\dfrac{d}{0.1178}\right)^{3/2}$

where d is the depth below still water level for any given horizontal distance x_N measured from the existing shoreline, both measured in metres.

Cutoff depth $d_c = 1.75\ H_s = 1.75\ (2.0) = 3.5$ m

Intersection depth $d_i = 3\ H_s = 3\ (2.0) = 6.0$ m

The equilibrium profile, down to a cutoff depth d_c, may be approximated by the following equation :

$$l_R = 0.7301\, l_N$$

$$x_R - Y = 0.7301\ x_N$$

$$x_R - 30.0 = 0.7301\left(\frac{d}{0.1178}\right)^{3/2}$$

$$d = 0.1178\left(\frac{x_R - 30.0}{0.7301}\right)^{2/3}$$

$$d = 0.1453\,(x_R - 30.0)^{2/3}$$

or $$x_R = \left(\frac{d}{0.1453}\right)^{3/2} + 30.0$$

At cutoff depth d_c, $x_R = 148.2$ m

$x_N = 162.0$ m

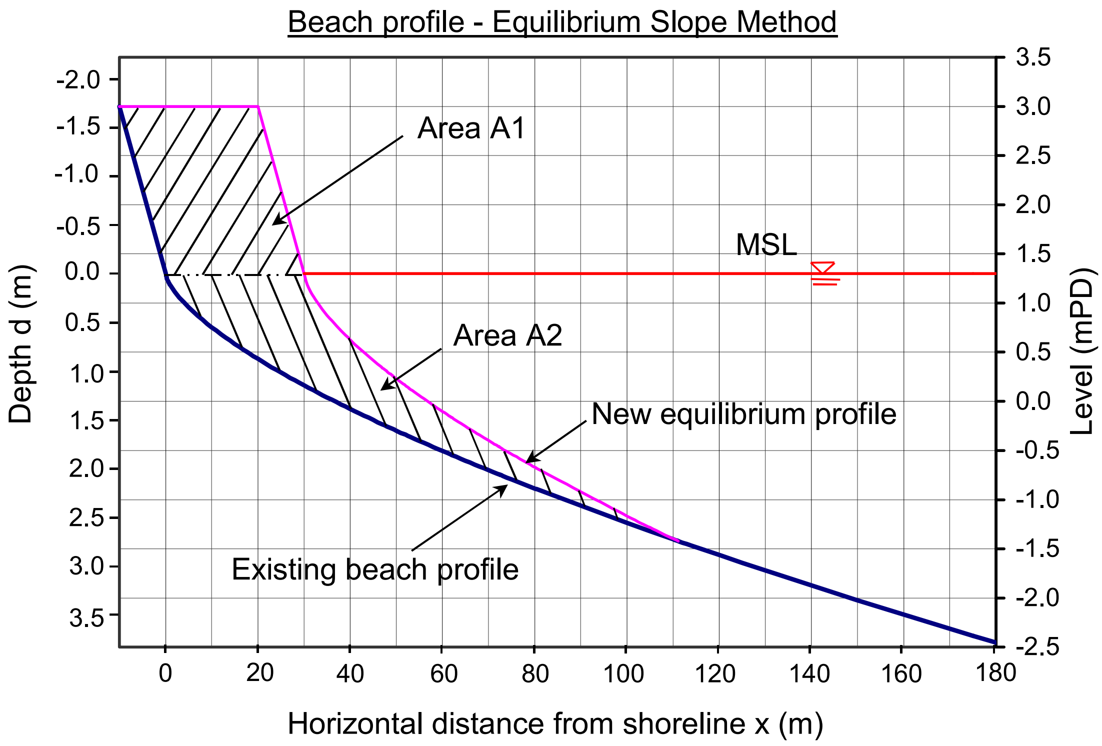
Since $x_N > x_R$, the recharged beach profile intersects the existing beach profile before the cutoff depth.

The volume of fill required is estimated from the shaded area in the figure below :

= A1 + A2

= 30.0 × 1.7 + 47.6

= 98.6 m^3/m



(3) Overfill Ratio

By Krumbein and James Method :

Given further that $\phi_{N\_84} = -\log_2 0.2 = 2.32$

$$\phi_{N\_16} = -\log_2 0.4 = 1.32$$

$$\phi_{R\_84} = -\log_2 0.45 = 1.15$$

$$\phi_{R\_16} = -\log_2 0.55 = 0.86$$

where ϕ_{N_84} and ϕ_{R_84} are the particle sizes that are exceeded by 84% by dry weight of the native and recharged sand samples respectively in phi unit. Similarly, ϕ_{N_16} and ϕ_{R_16} are the particle sizes that are exceeded by 16% by dry weight of the sand samples in phi unit.

Therefore,

$$M_{\phi N} = (\phi_{N\_84} + \phi_{N\_16})/2 = (2.32 + 1.32)/2 = 1.82$$

$$M_{\phi R} = (\phi_{R\_84} + \phi_{R\_16})/2 = (1.15 + 0.86)/2 = 1.01$$

$$\sigma_{\phi N} = (\phi_{N\_84} - \phi_{N\_16})/2 = (2.32 - 1.32)/2 = 0.50$$

$$\sigma_{\phi R} = (\phi_{R\_84} - \phi_{R\_16})/2 = (1.15 - 0.86)/2 = 0.15$$

$$\therefore \; M_{\phi R} < M_{\phi N} \;\&\; \sigma_{\phi R} < \sigma_{\phi N}$$

⇒ The distribution cannot be matched but the fill should be stable. Scouring of native material fronting toe of fill may be induced. The overfill ratio is re-estimated by the Dean Method.

By Dean Method :

The standard deviation of the fill is found to be 0.05 mm, determined from sieve tests.

Size in phi unit = $-\log_2 D$

$$\mu_n^{'} = \frac{\mu_n}{\sigma_b} = \frac{-\log_2 0.3}{-\log_2 0.05} = \frac{1.74}{4.32} = 0.40$$

$$\mu_b^{'} = \frac{\mu_b}{\sigma_b} = \frac{-\log_2 0.5}{-\log_2 0.05} = \frac{1.00}{4.32} = 0.23$$

From Figure A3, the overfill ratio is equal to 1.

(4) Sand Quantity

The average of the fill volume estimated by the equilibrium profile method and the equilibrium slope method is (125.8 + 98.6)/2 = 112.2 m^3/m. Since the overfill ratio is 1, the sand quantity is therefore equal to 112.2 m^3/m.

It should be noted that loss of fill may occur due to actions of waves and currents during fill placement. The actual fill quantity required may be more than the above-calculated quantity.

B.2 CONSTRUCTION PROFILE AND EQUILIBRIUM PROFILE

Reference Sections 5.3 and 5.4.

Given further that

Mean low lower water level = +0.5 mPD

Extreme water level = +2.9 mPD

Find

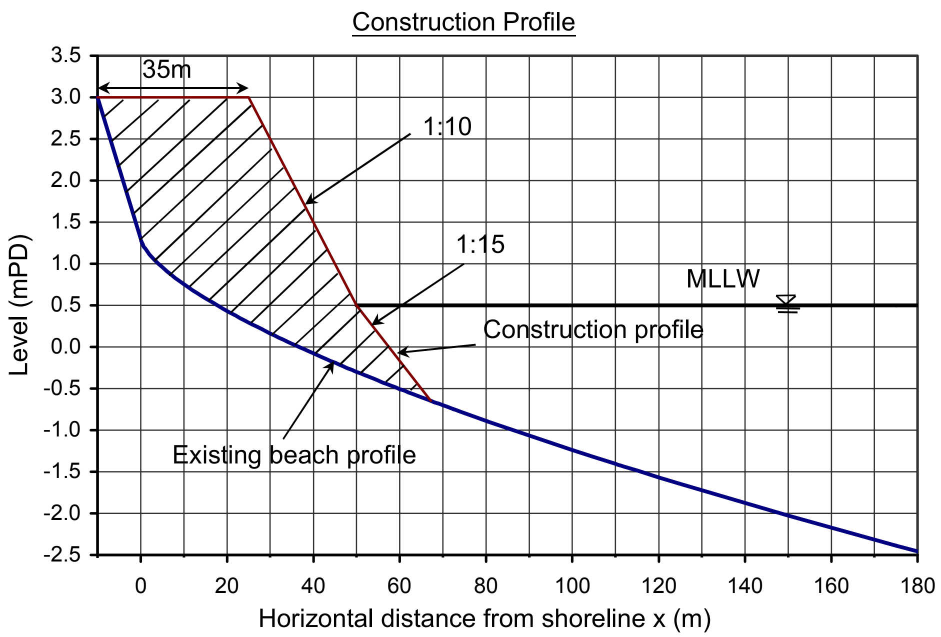
Determine the construction profile based on the fill quantity computed in Example B.1 and re-estimate the equilibrium profile.

Solution

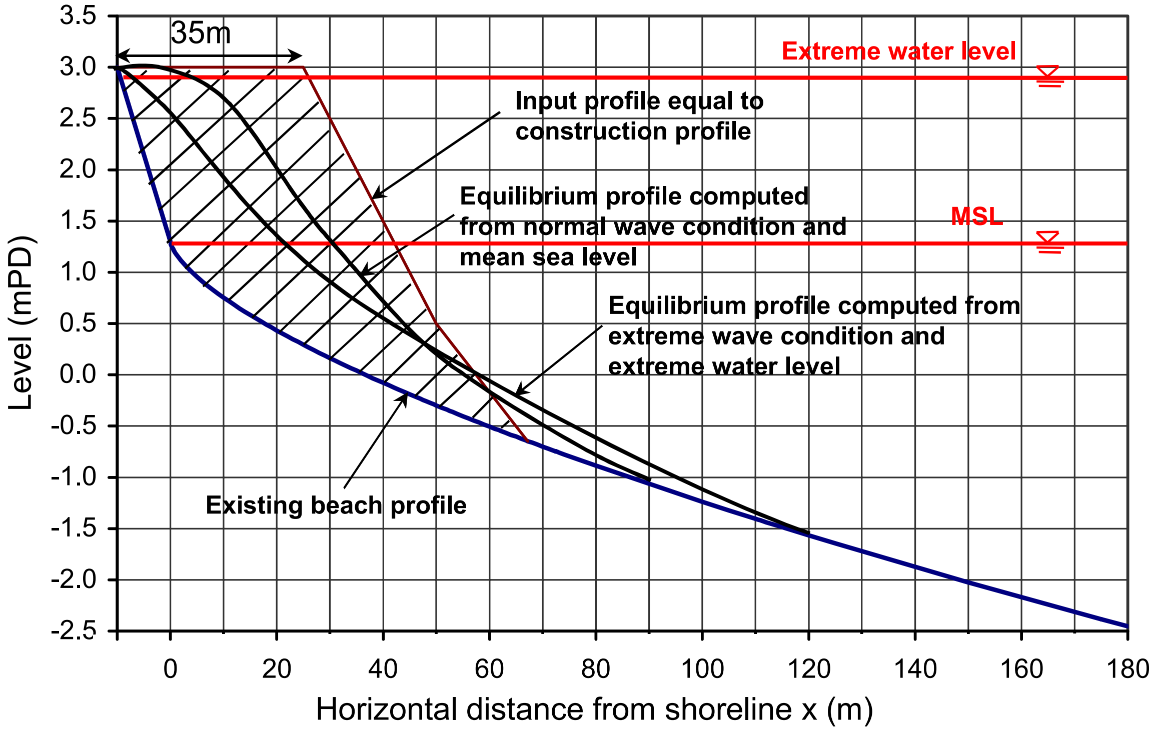
Estimated sand quantity = 112.2 m^3/m from Example B.1.

Try a berm width of construction profile = 35 m and assume upper slope = 1:10, lower slope = 1:15 for $D_{50} = 0.3 \sim 0.5\,\text{mm}$.

Based on the above information, the construction profile is plotted against the existing beach profile as shown in the following graph. From the graph, the re-calculated sand quantity is equal to 115 m^3/m. Therefore, take this as the construction profile.



The equilibrium profile from empirical formulae in Example B.1 is an initial estimate for computing the sand quantity. A more precise, efficient estimate of the equilibrium profile can be carried out by means of cross-shore transport modelling, using the construction profile as a starting profile together with relevant extreme/normal wave conditions and water levels. Details of input requirements should be made to the user's manual of the modelling software. Diagrammatic sketch of the computation results is shown in the following graph.



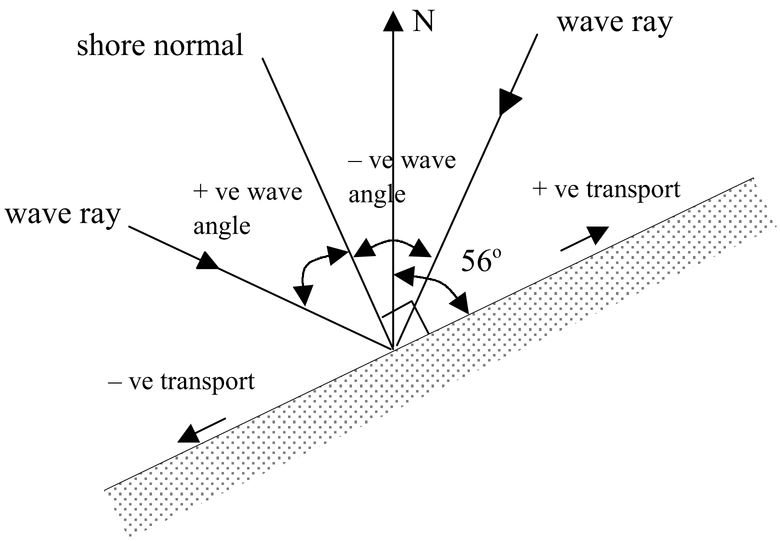
The above computed equilibrium profile under extreme condition may be used as input profile to re-compute the equilibrium profile under normal condition. Alternatively, the computed equilibrium profile under normal condition may be used as the input profile to re-compute the equilibrium profile under extreme condition. These steps have to be re-iterated to assess the final equilibrium profile of the beach.

B.3 NET LONGSHORE TRANSPORT

Reference Sections 5.5.1 and 5.5.2.

Given

The following table gives the fictitious wave height, period, incident angle and duration of occurrence at a shore running 56^o from the North.



Wave height at breaker line H_b (m)	Yearly occurrence percentage (%)	Period T (s)	Incident wave bearing (º)
calm (<0.06)	65.2	-	-
0.06	3.0	0.86	353.7
0.24	3.8	1.92	351.6
0.34	0.3	2.60	341.7
0.07	3.7	0.91	16.7
0.20	5.9	2.07	1.9
0.34	0.4	2.80	349.6
0.08	1.9	3.30	269.3
0.19	0.3	4.00	272.6
0.09	1.5	1.24	256.1
0.23	4.2	2.84	270.2
0.36	0.1	3.90	276.4
0.11	2.1	1.13	259.2
0.29	1.2	2.66	272.9
0.37	0.1	3.70	279.4
0.09	1.5	1.11	281.7
0.28	1.1	2.59	281.8
0.38	0.3	3.55	286.5
0.08	1.8	0.95	318.7
0.26	1.5	2.08	314.8
0.36	0.1	2.80	313.8
> 0.38	negligible	-	-

Find

Determine the net longshore transport rate and direction.

Solution

Using CERC longshore transport formula :

$$s_l = BH_b^2 c_b \sin(2\phi_b)$$

where s_l = longshore transport due to breaking wave (m^3/s)

B = a constant equal to about 0.025

H_b = wave height at breaker line (m)

ϕ_b = breaker angle with respect to shore normal (^{o})

c_b = wave velocity at breaker line (m/s)

$$= \frac{L_b}{T}$$

L_b = wavelength at breaking

$$= \frac{gT^2}{2\pi}\tanh\left(\frac{2\pi d_b}{L_b}\right)$$

d_b = depth at breaking

= $H_b/0.78$ (a simplified, approximate assumption)

Wave Condition i	H_b (m)	ϕ_b relative to shore normal (°)	T (s)	$d_b = \frac{H_b}{0.78}$ (m)	$L_b = \frac{gT^2}{2\pi}\tanh(\frac{2\pi d_b}{L_b})$ (m)	$c_b = L_b/T$ (m/s)	s_l (m³/s)
1	0.06	–27.7	0.86	0.08	0.69	0.81	–0.00006
2	0.24	–25.6	1.92	0.31	3.15	1.64	–0.00184
3	0.34	–15.7	2.60	0.44	5.14	1.98	–0.00297
4	0.07	–50.7	0.91	0.09	0.79	0.87	–0.00010
5	0.20	–35.9	2.07	0.26	3.15	1.52	–0.00145
6	0.34	–23.6	2.80	0.44	5.57	1.99	–0.00421
7	0.08	56.8	3.30	0.10	3.29	1.00	0.00015
8	0.19	53.5	4.00	0.24	6.12	1.53	0.00132
9	0.09	70.0	1.24	0.12	1.25	1.01	0.00013
10	0.23	55.9	2.84	0.29	4.71	1.66	0.00204
11	0.36	49.7	3.90	0.46	8.13	2.08	0.00666
12	0.11	66.9	1.13	0.14	1.23	1.09	0.00024
13	0.29	53.2	2.66	0.37	4.90	1.84	0.00372
14	0.37	46.7	3.70	0.47	7.80	2.11	0.00720
15	0.09	44.4	1.11	0.12	1.11	1.00	0.00020
16	0.28	44.3	2.59	0.36	4.69	1.81	0.00354
17	0.38	39.6	3.55	0.49	7.56	2.13	0.00755
18	0.08	7.4	0.95	0.10	0.88	0.93	0.00004
19	0.26	11.3	2.08	0.33	3.57	1.71	0.00111
20	0.36	12.3	2.80	0.46	5.72	2.04	0.00275

The net transport is given by :

$$S_l = \sum_{i}^{n} s_{l\,i} \cdot p_i$$

where S_l = net longshore transport (m^3/year)

s_{li} = longshore transport due to wave condition i

p_i = duration of occurrence of wave condition i in a year (s)

= % of time per year × 31,536,000 s

Wave condition i	S_{li} (m^3/s)	Occurrence per year (%)	p_i (s)	S_l (m^3/year)
1	–0.00006	3.0	946,080	–57
2	–0.00184	3.8	1,198,368	–2,202
3	–0.00297	0.3	94,608	–281
4	–0.00010	3.7	1,166,832	–122
5	–0.00145	5.9	1,860,624	–2,689
6	–0.00421	0.4	126,144	–532
7	0.00015	1.9	599,184	88
8	0.00132	0.3	94,608	125
9	0.00013	1.5	473,040	62
10	0.00204	4.2	1,324,512	2,700
11	0.00666	0.1	31,536	210
12	0.00024	2.1	662,256	158
13	0.00372	1.2	378,432	1,407
14	0.00720	0.1	31,536	227
15	0.00020	1.5	473,040	95
16	0.00354	1.1	346,896	1,230
17	0.00755	0.3	94,608	714
18	0.00004	1.8	567,648	21
19	0.00111	1.5	473,040	525
20	0.00275	0.1	31,536	87
			$\sum s_{li} =$	1,766

The total positive longshore transport quantity is +7,648 m^3/year and the total negative longshore transport quantity is –5,882 m^3/year. Therefore, the net longshore transport quantity is +1,766 m^3/year. The positive sign means that the net longshore transport direction is running towards the N 56^o E direction.

(Note: Other longshore transport formulae may also be used to compute the longshore transport.)

B.4 EQUILIBRIUM SHORELINE ORIENTATION

Reference Section 5.5.3.

Given

The same data as given in Example B.3.

Find

The resultant wave angle of incoming waves and equilibrium shoreline orientation.

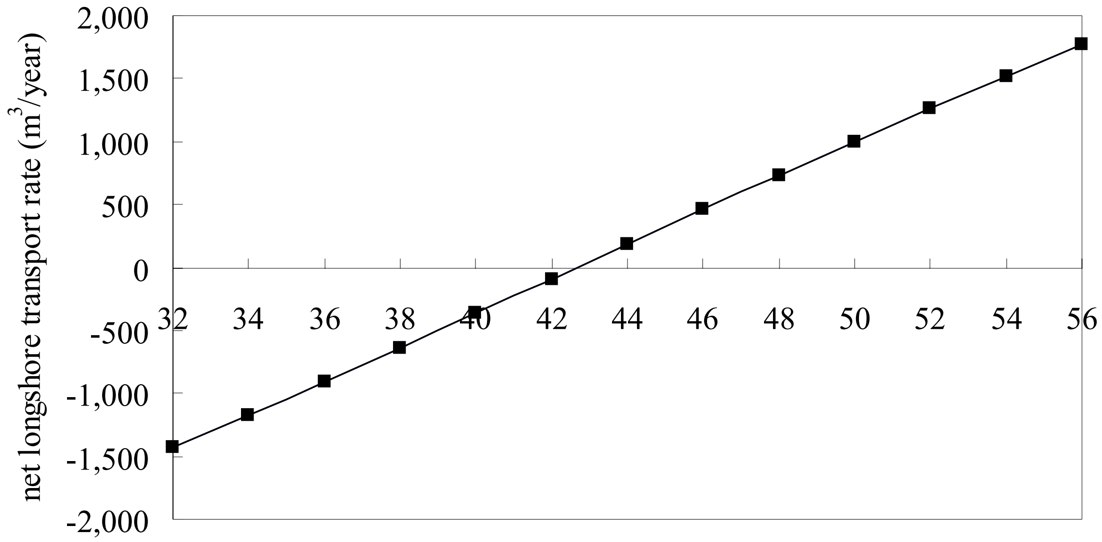
Solution

By following the approach in Example B.3 and assuming the wave height and wave angle at breaker line remain unchanged, the net longshore transportation rates for shoreline orientation varying from 32° to 56° N were determined in the following table:

Shoreline orientation (degree N)	Net longshore transport (m^3/year)
32	–1,428
34	–1,169
36	–905
38	–636
40	–363
42	–89
44	185
46	459
48	730
50	998
52	1,260
54	1,517
56	1,766

The longshore transport rates at different shoreline orientations are plotted against the shoreline orientations in the following graph.

Variation of net longshore transport with shoreline orientation



shoreline orientation (degree N)

From the graph, it can be observed that the net transport rate will be equal to zero m^3/year when the shoreline orientation is about 43°. The direction of net longshore transport calculated in Example B.3 indicates that the resultant wave angle should be on the left hand side of the shore normal. This also means that the resultant wave angle of the incoming waves given in Example B.3 is 133° as it should be normal to the equilibrium orientation.

B.5 EQUILIBRIUM PLAN FORM

Reference Section 5.5.5.

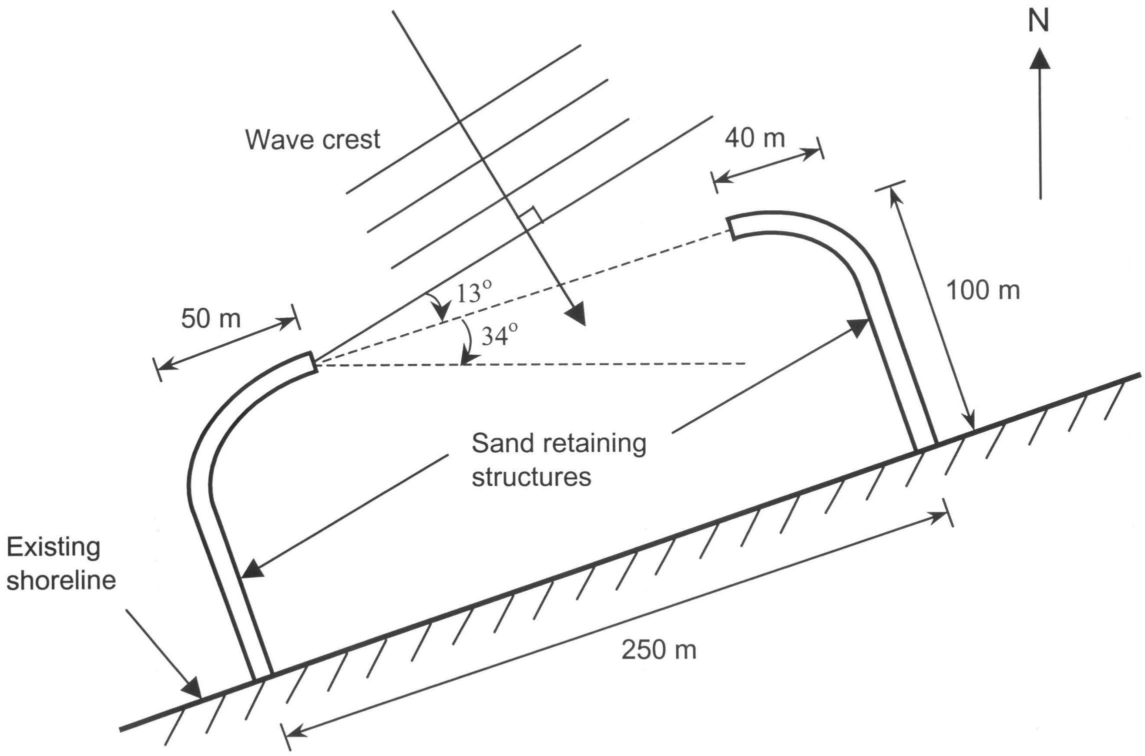
Given

The same data as given in Example B.3. The resultant incoming wave angle of the wave climate is 133° from the north.

Find

Determine the equilibrium beach plan form and layout of sand retaining structures at a shore running 56° from the north. The required minimum beach width at mean sea level is 30 m. The required beach length is about 250 m.

Solution



The method given in Section 5.5.5(3) and Figure 15 is applied to illustrate the design principle. The determination of the equilibrium plan form of the beach and the layout of sand retaining structures is a trial and error process. An initial assumed R_o and structure layout is required, followed by subsequent refinement until the minimum beach width requirement is achieved. The layout as shown above is investigated in this worked example.

Trial 1

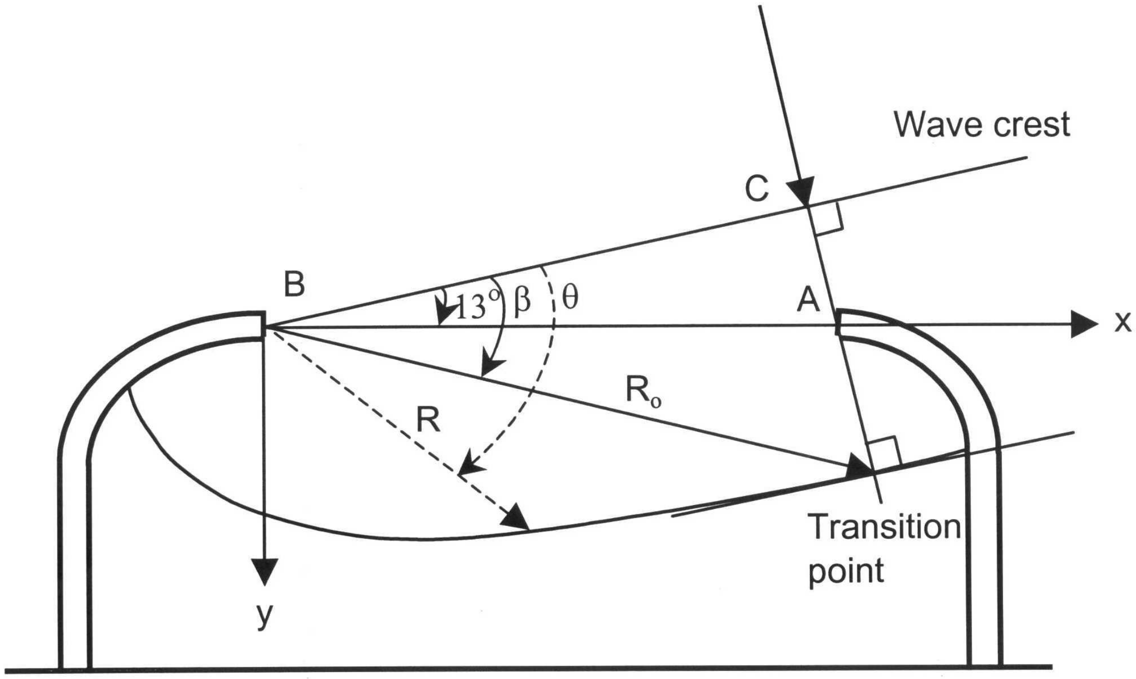
Assume $R_o = 175.0$ m

$AB = 250.0 - 50.0 - 40.0 = 160.0$ m

$BC = 160.0 \cos 13^{o} = 155.9$ m

$$\cos\beta = \frac{BC}{R_o} = \frac{155.9}{175.0} = 0.89$$

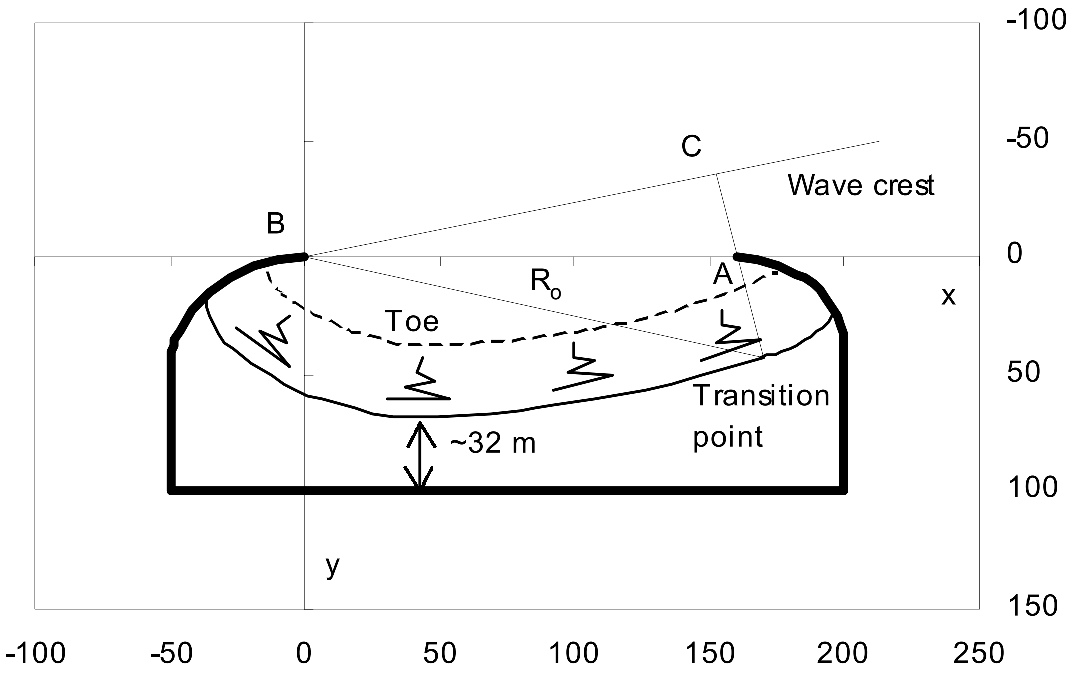
$\beta = 27.0^{o}$



Compute the equilibrium plan form.

R_o = 175.0 m and β = 27.0°
$x = R \cos(\theta - 13°)$
$y = R \sin(\theta - 13°)$

θ (degree)	R/R_o [see Figure 15]	R (m)	x (m)	y (m)
27	1.00	175.0	169.8	42.4
37	0.79	138.4	126.4	56.3
47	0.64	111.3	92.2	62.3
57	0.54	95.2	68.4	66.1
67	0.48	84.0	49.3	67.9
77	0.43	75.0	32.8	67.4
87	0.38	66.3	18.3	63.8
97	0.35	60.7	6.3	60.4
107	0.32	56.4	–4.0	56.3
117	0.30	52.1	–12.6	50.5
127	0.28	49.4	–20.1	45.1
137	0.27	47.4	–26.5	39.3
147	0.26	45.4	–31.6	32.7
157	0.25	43.3	–35.0	25.4
167	0.23	41.0	–36.9	18.0
170	0.23	40.3	–37.2	15.5



For the portion of the beach beyond the transition point, i.e., with angle $\theta < \beta$, Figure 15 provides no information about the arc ratio of the equilbrium bay shape. But as the waves will diffract around the upcoast headland, this portion of the beach could be assumed to be a circular arc with its centre located at point A and radius equal to the distance between the transition point and point A.

The above plan form with a maximum value of y = 67.9 m can satisfy the minimum beach width requirement of 30 m. Additional trials can be made if the exact minimum width of 30 m is required.

In addition, the design should ensure the toe of the beach lies within the sand retaining structures. Otherwise, sand loss may occur. The location of the toe may be estimated from the slope of equilibrium beach profile; methods to determine the equilibrium beach profile are shown in Examples B.1 and B.2.

The sand quantity required to achieve the minimum width of 30 m may then be estimated from the equilibrium plan form and the equilibrium profile within the sand retaining structures. A number of trials may need to be carried out to determine the optimal design layout.