APPENDIX D

WORKED EXAMPLES

CONTENTS

APPENDIX D WORKED EXAMPLES

D.1 Example 1 – Stability Analysis for Leading Edge of Reclamation

Given

1. A reclamation site has marine deposits of average undrained shear strength 5.0 kPa and thickness of 10.0 m with a water depth of 10.0 m.
2. Unit weight of reclamation fill is 19.0 kN/m^3.
3. The factor of safety for the leading edge should not be less than 1.2.

Task

Determine the minimum length of the leading edge if the first layer of fill to be placed is 3.0 m thick.

Solution

Equation 3.3 may be applied for preliminary analysis. The length of the leading edge of the reclamation is taken as the length from the toe to the crest of the embankment L_e in the equation.

The minimum length of leading edge can therefore be calculated using Equation 3.3 as below:

Undrained shear strength, c_u	=	5.0 kPa
Submerged unit weight, $\gamma_{(submerged)}$	=	19.0-10.1 = 8.9 kN/m^3
		(unit weight of seawater is 10.1 kN/m^3)
Mud thickness, D	=	10.0 m
Fill thickness, H	=	3.0 m
Required FOS, F	=	1.2

Assume the friction angle of the fill is 30°, then

$$K_a = \frac{1-\sin 30^\circ}{1+\sin 30^\circ}$$

$$= 0.3$$

$$\alpha = -\frac{0.3(8.9)(3^2)(1.2)}{2(5.0)(L_e)} = \frac{-2.8836}{L_e}$$

Therefore,

$$\frac{(1.2)(8.9)(3.0)}{5.0} = 4 + \frac{(1 + \frac{-2.8836}{L_e})L_e}{10.0}$$

$$L_e = 27.0 \text{ m}$$

For detailed analysis of stability, reference should be made to Section 3.1.3.

D.2 Example 2 – Estimation of Residual Settlement with Surcharge Preloading

Given

1. The loading variations of a small reclamation project, which involve filling to final formation level, application of surcharge preloading, removal of surcharge preloading and application of future imposed load, are shown in Figure D1.

2. Vertical drains are installed before fill placement and the programme of reclamation is assumed as follows:

Completion of vertical drain installation:	commencement of filling
Completion of filling to final formation level:	the end of the 9th month
Filling of surcharge preloading:	in the 10th month
Removal of surcharge preloading:	end of the 22nd month
(removal up to final formation level +4.5 mPD)	
Application of future imposed load:	in the 24th month

Note : The time in the above programme is measured from commencement of filling.

3. Design data of the reclamation is shown below:

Table D2.1

Reclamation Profile	
Seabed level	-8.0 mPD
Mean sea level	+1.3 mPD
Final formation level	+4.5 mPD
Surcharge preloading thickness	5.0 m
Marine deposits data	
Thickness, H	10.0 m
Drainage condition	Double drainage
Unit weight, γ_{MD}	16.0 kN/m^3
Coefficient of consolidation (vertical), c_v	1.5 m^2/yr
Coefficient of consolidation (horizontal), c_h	1.5 m^2/yr
Compression ratio, CR	0.29
Recompression ratio, RR	0.06
Coefficient of secondary consolidation, $C_{\alpha\varepsilon}$	0.005
Fill data	
Unit weight, γ_{fill}	19.0 kN/m^2
Logarithmic creep compression rate, α	1.0%

Vertical drains data	
Type	Prefabricated band drains (100 mm wide x 5 mm thick)
Spacing	1.5 m
Pattern	Triangular and full depth
Future imposed load	
Future imposed load	20.0 kPa

Task

Carry out preliminary assessment of the following:

1. Final primary consolidation settlement due to reclamation fill and future imposed load.
2. Primary consolidation settlement occurred at end of 22^{nd} month.
3. Secondary consolidation of marine deposit.
4. Creep of reclamation fill.
5. Residual settlement at end of 22^{nd} month.

Approach

Preloading by surcharge load with band drain installation is applied to accelerate the consolidation settlement. The marine deposit is subject to time-dependent loading as shown in Figure D1. The loading due to reclamation fill is taken to be increased linearly and the non-uniform loading pattern is schematised as uniform loading pattern.

The marine deposit is divided into 10 sub-layers in the calculation and the degree of consolidation is calculated for each sub-layer. The primary consolidation settlement is obtained by summing up the settlement of each sub-layer.

Band drains are installed before commencement of filling and their function is assumed normal throughout the reclamation period and the surcharging period. The compression ratio, CR, is assumed to be constant for the whole marine deposit stratum and also remain constant in the process of primary consolidation. If the soil is variable, the depth should be represented by soil layers with different CR with reference to site investigation results.

N.B.

a. Only settlements in marine deposit and reclamation fill are considered and settlement in other soil stratum is neglected. In real case, settlement of all soil strata, including the alluvium clay layer, should also be considered.

b. The designer shall decide the most appropriate schematization for the loading sequence.
c. Alluvium sand is assumed underlying the marine deposit. As such, double drainage condition can be assumed.

Settlement estimation

(1) Final primary consolidation settlement due to reclamation fill and future imposed load

The final formation level of the reclamation is +4.5 mPD after removal of surcharge preloading. Taking into account the settlement, which is initially assumed to be 3.0 m (it should be adjusted based on subsequent calculated settlement), the total effective stress increment due to the loading of reclamation fill is given by:

Depth of fill below MSL = +1.3 mPD - (-8.0 mPD) + 3.0 m = 12.3 m
Depth of fill above MSL = +4.5 mPD - 1.3 mPD = 3.2 m

$$\Delta\sigma_{v(\text{fill})} = 12.3(19.0 - 10.1) + 3.2(19.0) = 170.3 \text{ kN/m}^2$$

$$\Delta\sigma_{v(imposed\ \ load)} = 20.0 \text{ kN/m}^2$$

The primary consolidation settlement due to the loading of reclamation fill and future imposed load of each sub-layer (without surcharge preloading) is calculated as follows:

Table D2.2

Sub-layer	z (mPD)	Initial effective stress (kN/m^2)	Total effective stress increment (kN/m^2)	Settlement (m)
1	-8.5	2.95	190.3	0.53
2	-9.5	8.85	190.3	0.39
3	-10.5	14.75	190.3	0.33
4	-11.5	20.65	190.3	0.29
5	-12.5	26.55	190.3	0.26
6	-13.5	32.45	190.3	0.24
7	-14.5	38.35	190.3	0.22
8	-15.5	44.25	190.3	0.21
9	-16.5	50.15	190.3	0.20
10	-17.5	56.05	190.3	0.19
			Total	**2.87**

The final primary consolidation settlement due to the loading of reclamation fill and future imposed load, $S_{p(final)}$, is estimated to be 2.87 m. This amount is close to the initial assumed value of 3 m and therefore recalculation is not made in this example.

(2) Primary consolidation settlement occurred at end of 22nd month

(2a) Degree of consolidation due to the loading of reclamation fill (with vertical drains) at the end of the 9.5th month (see Figure D1)

Consolidation due to vertical drainage alone

Applying the correction for construction time (see Section 4.3.3) and taking into account the construction time for filling the surcharge mound,

$$t = 10 - \frac{9}{2} - \frac{1}{2} = 5\,\text{months} = 0.42\text{ years}$$

$$T_v = \frac{c_v t}{d^2} = \frac{1.5 \times 0.42}{5^2} = 0.025$$

(for double drainage, d = 10.0/2 = 5.0 m)

Equation B2a is applied to calculate the degree of consolidation for each sub-layer of the marine deposit as below:

Table D2.3

m	M	$\frac{2}{M}\sin\left(\frac{Mz}{H_{dr}}\right)\exp(-M^2T_v)$ with z (m) =									
		0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5
0	0.5π	0.187	0.543	0.846	1.067	1.182	1.182	1.067	0.846	0.543	0.187
1	1.5π	0.111	0.241	0.172	-0.038	-0.217	-0.217	-0.038	0.172	0.241	0.111
2	2.5π	0.039	0.039	-0.039	-0.039	0.039	0.039	-0.039	-0.039	0.039	0.039
3	3.5π	0.008	-0.001	-0.006	0.009	-0.004	-0.004	0.009	-0.006	-0.001	0.008
	sum	0.344	0.821	0.974	0.999	1.000	1.000	0.999	0.974	0.821	0.344
	ui	0.656	0.179	0.026	0.001	0.000	0.000	0.001	0.026	0.179	0.656

Consolidation due to horizontal drainage alone

$$t = 0.42\text{ year}$$

$$\text{D} = 1.05 \times 1.5 = 1.58\,\text{m}$$

$$T_h = \frac{c_h t}{D^2} = \frac{1.5 \times 0.42}{1.58^2} = 0.25$$

$$d' = \frac{2(5+100)}{\pi} = 66.8 \text{ mm}$$

$$n = \frac{D}{d'} = 23.7$$

$$F(n) = \frac{n^2}{n^2-1}\ln n - \frac{3n^2-1}{4n^2}$$
$$= 2.42$$

From Equation B3,

$$U_h = 1 - \exp[\frac{-8T_h}{F(n)}]$$
$$= 1 - \exp[\frac{-8 \times 0.25}{2.42}]$$
$$= 0.56$$

Consolidation under combined vertical and horizontal drainage

From Equation B4, $U_f = U_h + U_v - U_h U_v$, the degree of consolidation at each sub-layer is calculated as below:

Table D2.4

Layer	z (mPD)	ui
1	-8.5	0.849
2	-9.5	0.639
3	-10.5	0.571
4	-11.5	0.561
5	-12.5	0.560
6	-13.5	0.560
7	-14.5	0.561
8	-15.5	0.571
9	-16.5	0.639
10	-17.5	0.849

(2b) Effective stress increment at the end of the 9.5th month (see Figure D1)

The marine deposit is subject to different degree of consolidation at different depth. The calculation of the primary consolidation settlement for the first sub-layer is illustrated as an example as follows.

The initial effective stress at the center of the first marine deposit sub-layer is given by

$$\sigma_{v0,z=0.5m}' = 0.5(16.0 - 10.1) = 2.95\ \text{kN/m}^2$$

The first estimate of settlement occurs at end of 9.5th month is taken to be 1.9 m (it should be adjusted based on subsequent calculated settlement). Taking into account the effect of settlement in the change of effective stress increment (larger portion of reclamation fill is submerged below seawater level), the effective stress increment due to the loading of reclamation fill at the centre of the first sub-layer is given by:

Depth of fill below MSL = +1.3 mPD - (-8.0 mPD) + 1.9 m = 11.2 m
Depth of fill above MSL = +4.5 mPD - 1.3 mPD - 1.9 m = 1.3 m

$$\begin{aligned}\Delta\sigma_{v(\text{fill})} &= [11.2\times(19.0-10.1)+1.3\times19.0]\times0.849 \quad \text{(See Table D2.4)}\\ &= 124.4\times0.849\\ &= 105.6\ \text{kN/m}^2\end{aligned}$$

From Equation 4.1a, the primary consolidation settlement in the first sub-layer (H = 1.0 m for each sub-layer) due to the loading of reclamation fill is given by:

$$\begin{aligned}S_{p(\text{fill}),\ \text{z=0.5m, 9.5th month}} &= HCR\log\left(\frac{\sigma_{v0}'+\Delta\sigma_{v(\text{fill}),\ \text{z=0.5m, 9.5th month}}}{\sigma_{v0}'}\right)\\ &= 1.0(0.29)\log\left(\frac{2.95+105.6}{2.95}\right)\\ &= 0.45\ \text{m}\end{aligned}$$

Check the magnitude of the primary consolidation settlement of the soil:

Table D2.5

Layer	z (mPD)	Initial effective stress (kN/m^2)	Effective stress increment (kN/m^2)	Settlement at the end of 9.5th month (m)
1	-8.5	2.95	105.6	0.45
2	-9.5	8.85	79.5	0.29
3	-10.5	14.75	71.1	0.22
4	-11.5	20.65	69.7	0.19
5	-12.5	26.55	69.7	0.16
6	-13.5	32.45	69.7	0.14
7	-14.5	38.35	69.7	0.13
8	-15.5	44.25	71.1	0.12
9	-16.5	50.15	79.5	0.12
10	-17.5	56.05	105.6	0.13
			Total	1.96

The primary consolidation settlement due to the loading of reclamation fill at the end of 9.5th month is estimated to be 1.96 m and is close to the initial estimate of 1.9 m. Therefore, recalculation of the effect of settlement on the change in effective stress increment is not carried out in this example.

(2c) The degree of primary consolidation due to reclamation fill and surcharge preloading (with vertical drains) at the time of surcharge removal (end of 22nd month)

Consolidation due to vertical drainage alone

Applying the correction for construction time (see Section 4.3.3),

$$t = 13 - \frac{1}{2} = 12.5 \text{ months} = 1.04 \text{ years}$$

$$T_v = \frac{c_v t}{d^2} = \frac{1.5 \times 1.04}{5.0^2} = 0.062$$

For rough estimation, Equation B2a is assumed valid to calculate the degree of consolidation for each sub-layer of the marine deposit. They are calculated as below:

Table D2.6

m	M	$\frac{2}{M}\sin\left(\frac{Mz}{H_{dr}}\right)\exp(-M^2T_v)$ with z (m) =									
		0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5
0	0.5π	0.171	0.496	0.773	0.974	1.079	1.079	0.974	0.773	0.496	0.171
1	1.5π	0.049	0.106	0.076	-0.017	-0.095	-0.095	-0.017	0.076	0.106	0.049
2	2.5π	0.004	0.004	-0.004	-0.004	0.004	0.004	-0.004	-0.004	0.004	0.004
3	3.5π	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	sum	0.224	0.606	0.844	0.953	0.988	0.988	0.953	0.844	0.606	0.224
	ui	0.776	0.394	0.156	0.047	0.012	0.012	0.047	0.156	0.394	0.776

Note :

Strictly speaking, Equation B2a is not valid as the excess pore water pressure is not constant throughout the marine deposit layer. The excess pore water pressure, u_e, at depth z after time t is given by:

$$u_e = \sum_{n=1}^{n=\infty}\left(\frac{1}{d}\int_0^{2d} u_i \sin\frac{n\pi z}{2d}dz\right)\left(\sin\frac{n\pi z}{2d}\right)\exp\left(-\frac{n^2\pi^2 c_v t}{4d^2}\right)$$

Alternatively, the method developed by M. Carter for Macau International Airport can be adopted to determine the time curve for the degree of consolidation. For details, see pp 35-40 in Transactions, Volume 3, Number 1, the Hong Kong Institution of Engineers.

Consolidation due to horizontal drainage alone

$t = 1.05$ year

$\mathrm{D} = 1.05 \times 1.5 = 1.58$ m

$$T_h = \frac{c_h t}{D^2} = \frac{1.5 \times 1.05}{1.58^2} = 0.63$$

$$d' = \frac{2(5+100)}{\pi} = 66.8\,\text{mm}$$

$$n = \frac{D}{d'} = 23.7$$

$$F(n) = \frac{n^2}{n^2-1}\ln n - \frac{3n^2-1}{4n^2}$$

$$= 2.42$$

From Equation B3,

$$U_h = 1 - \exp[\frac{-8T_h}{F(n)}]$$
$$= 1 - \exp[\frac{-8 \times 0.63}{2.42}]$$
$$= 0.88$$

Consolidation under combined vertical and horizontal drainage

From Equation B4, $U_f = U_h + U_v - U_h U_v$, the degree of consolidation at each sub-layer is calculated as below:

Table D2.7

Layer	z (mPD)	ui
1	-8.5	0.973
2	-9.5	0.927
3	-10.5	0.899
4	-11.5	0.886
5	-12.5	0.881
6	-13.5	0.881
7	-14.5	0.886
8	-15.5	0.899
9	-16.5	0.927
10	-17.5	0.973

(2d) Primary consolidation settlement due to reclamation fill and surcharge preloading (with vertical drain) at the time just immediately after surcharge removal (end of 22nd month)

The first estimate of settlement occurs at end of 22nd month is taken to be 3.0 m (it should be adjusted based on subsequent calculated settlement). The effective stress increment due to the loading of reclamation fill at the centre of the first sub-layer is given by:

Depth of fill below MSL = +1.3 mPD - (-8.0 mPD) + 3.0 m = 12.3 m
Depth of fill above MSL = 5.0 m + 4.5 mPD - 1.3 mPD – 3.0 m = 5.2 m

The effective stress increment if all pore pressure is dissipated at end of 22nd month:

$$\Delta\sigma_v = 12.3(19.0 - 10.1) + 5.2(19.0) = 208.3 \text{ kN/m}^2$$

Effective stress increment at end of 9.5 months = 105.6 kN/m^2 (see Table D2.5)

As only 97.3% pore water pressure is dissipated at end of 22nd month (see Table D2.7), therefore

$$\Delta\sigma_{v,\text{ z=0.5m, 22th month}} = 105.6 + (208.3 - 105.6) \times 0.973 = 205.5\ \text{kN/m}^2$$

From Equation 4.1a, the primary consolidation settlement in the first layer (H = 1.0 m for each sub-layer) due to the loading of reclamation fill and surcharge preloading (with vertical drains) is given by:

$$\begin{aligned} S_{p,\text{ z=0.5m, 22th month}} &= HCR\log\left(\frac{\sigma_{v0}' + \Delta\sigma_{v,\text{ z=0.5m, 22nd month}}}{\sigma_{v0}'}\right) \\ &= 1.0(0.29)\log\left(\frac{2.95 + 205.5}{2.95}\right) \\ &= 0.54\ \text{m} \end{aligned}$$

The settlement for the primary consolidation settlement due to reclamation fill and surcharge preloading (with vertical drains) at the time of surcharge removal (end of the 22nd month) for all the sub-layers are calculated as follows:

Table D2.8

Sub-layer	z (mPD)	Initial effective stress (kN/m^2)	Effective stress increment (kN/m^2)	Settlement at the end of 22nd month (m)
1	-8.5	2.95	205.5	0.54
2	-9.5	8.85	198.9	0.40
3	-10.5	14.75	194.4	0.33
4	-11.5	20.65	192.5	0.29
5	-12.5	26.55	191.9	0.27
6	-13.5	32.45	191.9	0.24
7	-14.5	38.35	192.5	0.23
8	-15.5	44.25	194.4	0.21
9	-16.5	50.15	198.9	0.20
10	-17.5	56.05	205.5	0.19
			Total	**2.90**

The total consolidation settlement due to reclamation fill and surcharge preloading (with vertical drains) at the end of the 22nd month is estimated to be 2.9 m.

Note : The settlement at the end of the 22nd month is equal to 2.9 m which is very close to the initial estimate of 3.0 m. Further recalculation is therefore not required in this example.

(2e) Residual settlement due to primary consolidation

As the effective stress increment in the marine deposit is greater than that due to the future loading, the marine deposit is considered over-consolidated. A rebound will occur after removal of surcharge loading. The change in settlement will then follow recompression curve when the future imposed load (20.0 kN/m^2) is applied on the reclamation.

Effective stress due to self weight of marine mud (at centre of soil layers)
= 5.0 x (16.0 - 10.1) = 29.5 kN/m^2

Depth of fill below MSL = +1.3 mPD - (-8.0 mPD) + 2.9 m = 12.2 m
Depth of fill above MSL = +4.5 mPD - 1.3 mPD = 3.2 m
(Surcharge removed to formation level +4.5 mPD only)

Effective stress due to fill
= 12.2 x (19.0 - 10.1) + 3.2 x 19.0 = 169.4 kN/m^2

Effective stress due to imposed load = 20.0 kN/m^2

The order of the maximum recompression, when the rebound is fully developed after a certain period of time, can be approximated by:

$$S_{\max,recompression} = H.RR.\log(\frac{\sigma_{v0}'+\Delta\sigma_v}{\sigma_{v0}'})$$
$$= 10.0(0.06)\log(\frac{29.5+169.4+20}{29.5+169.4})$$
$$= 25 \text{ mm}$$

The residual settlement due to primary consolidation is therefore equal to about 25 mm.

N.B. The future imposed load will be applied shortly (2 months later) after the removal of the surcharge loading. As a result, the rebound will not be fully developed. The order of recompression should be smaller than 25 mm in principle.

(3) Settlement due to secondary consolidation in marine deposit

The residual settlement due to secondary consolidation is:

$$S_{s,t(\text{residual})} = C_{\alpha\varepsilon} H.\log(\frac{t_2 - t_o}{t_1 - t_o})$$
$$= 0.005 \times 10.0 \times \log(\frac{50 - 9.5/12}{22/12 - 9.5/12})$$
$$= 84 \text{ mm}$$

(4) Settlement within reclamation fill

Due to the primary settlement, the thickness of the reclamation fill is taken to be 15.5m. The residual settlement of reclamation fill due to creep at end of 22nd month:

$$S_{c,t(\text{residual})} = H\alpha \log(\frac{50 - \frac{t_c}{2}}{t - \frac{t_c}{2}})$$
$$= 15.5 \times 0.01 \times \log(\frac{50 - 9/2/12}{22/12 - 9/2/12})$$
$$= 237 \text{ mm}$$

(5) Residual settlement at end of 22nd month

Total residual settlement at the time of removal of surcharge is estimated to be
= $S_{s,p(\text{residual})} + S_{s,t(\text{residual})} + S_{c,t(\text{residual})}$
= 25 + 84 + 237
~ 350 mm

D.3 Example 3 – Monitoring of Pore Pressure for Stability during Fill Placement

Given

A piezometer installed in the marine deposit layer revealed abnormalities during the fill placement works. Increase of excess pore water pressure was found larger than the predicted value as shown in Figure D2. Pause of filling works was considered necessary (see Section 5.3.2).

The dimensions of vertical drains were the same as those in Example D.2. The coefficient of consolidation (horizontal) was taken as 1.5m^2/year.

Task

Estimate the time required (t) for the excess pore water pressure to fall back to the normal magnitude.

Approach

(1) Define the abnormal excess pore pressure as the initial excess pore pressure (u_o).
(2) Define the target normal excess pore pressure (u_e).
(3) Determine the required degree of consolidation (U_h) for dissipation of excess pore pressure from u_o to u_e.
(4) Back calculate the time factor (T_h) from the required U_h.
(5) Obtain the actual time t required from T_h.

Solution

Initial excess pore pressure, u_0 = 20.0 kN/m^2 (see Figure D2)
Excess pore water pressure at time t, u_e = 14.9 kN/m^2 (see Figure D2)
Pore pressure to be dissipated, d_p = 20.0 – 14.9 = 5.1 kN/m^2

Neglecting vertical drainage, average degree of consolidation required to dissipate the above pore pressure is given by:

$$U_h = \frac{5.1}{20} = 0.255$$

The value of *F(n)* in Example D.2 was used because the dimensions and layout of vertical drains were the same. Therefore, *F(n)* = 2.42.

From Equation B3,

$$\begin{aligned} T_h &= -\frac{F(n)}{8}\ln(1-U_h) \\ &= -\frac{2.42}{8}\ln(1-0.255) \\ &= 0.09 \end{aligned}$$

Therefore,

$$t = \frac{D^2 T_h}{c_h} = \frac{2.1(0.09)}{1.5} = 0.13\text{ year}$$

i.e. Time required for the excess pore pressure to fall back to the normal magnitude is about $1\frac{2}{3}$ months.

LIST OF FIGURES